The Japanese Stock Rate of Return and Volatility :

A Comparison of Methods to Estimate Volatilities*

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abstract

This paper investigates the correlation between the volatility and the stock returns using daily data for Japan. In order to investigate this relationship, a nonparametric method is used to estimate the conditional variance (or volatility) of the stock returns and its partial derivatives with respect to the level of the stock returns.

Two important features are found. First, the conditional variance of the Japanese stock returns is found to depend negatively on the past level of the stock returns. Second, there may be a positive relationship between the expected stock returns and the volatility. Third, the volatility function is found to be a nonlinear function of the stock returns.

Keywords: stock returns, volatility, nonparametric method

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1 INTRODUCTION

Explaining the movement of an asset's volatility, which induces a changing risk premium of the asset, is one of the most important problems of modern

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financial theories. A natural hypothesis in an efficient market is that markets movements reflect in risk premium changes induced by their movements in their volatilities (see Pindyck [1984]). On the U.S. stock market crash of October 1987, the standard deviation of the daily returns increased from about 1% to almost 7%. As well as the U.S. market, the standard deviation of the *Nikkei 225* daily returns increased from about 1% to almost 5% in the few days around the large drop of February 1990 in the Japanese stock market. Changes in risk premia of this magnitude may have important effects on stock returns and thus the levels of stock prices.

One striking characteristic of the stock markets is the correlation of volatility with the level of stock returns. Volatility is typically higher after the stock market falls than after it rises, so stock returns are negatively correlated with future volatility (see Black [1976]). There are two popular explanations for this relationship between stock return volatility and stock returns. First, the 'leverage effect' posits that an increase in leverage caused by a decline of the firm's market value induces an increase in the stock return volatility (see Black [1976] and Christie [1982]). Second, the 'volatility feedback effect' argued by Pindyck [1984] and French, Schwert and Stambaugh [1987] suggests that a forecasted upward changes in stock return volatilities raise expected future stock returns and cause a decline of contemporaneous stock returns. The positive contemporaneous correlation between stock returns and stock return volatility at the firm level stands in contrast to the negative correlation between aggregate stock returns and aggregate stock return volatility (see Duffee [1995]).

In the U.S. stock market, a number of authors have examined the correlation of volatility with the level of stock returns. Christie [1982] found a negative relation between contemporaneous stock returns and changes in volatility, whose magnitude was too large to be attributed solely to the leverage effect.

Poterba and Summers [1986] argued that the volatility feedback could not be important because shocks to stock market volatility do not persist for long periods. However, French, Schwert and Stambaugh [1987] also found a significant positive relation between the expected risk premia and volatility, which they attribute to the volatility feedback. Campbell and Hentschel [1992] built a direct model of the volatility feedback and concluded that it could be important during periods of high volatility.

This paper examines the correlation of volatility with the level of the stock returns in the Japanese stock market, using three statistical approaches including a new method not used in the earlier works. In the first, an implied volatility regression (IVR) model is used to investigate this relationship (see Poterba and Summers [1986]). In this model, the implied volatility (IV) is obtained by equating the observed option prices to their theoretical values, given the observed option prices and values for several parameters in the option pricing model (see Latane and Rendleman [1976]). This approach is not a time-series one and depends on the particular functional form of the specific pricing system adopted, for example, the Black-Scholes model. Second, a time-series model such as a model from the autoregressive conditional heteroskedasticity (ARCH) class is used to estimate the volatility and to investigate the predicted relationship (see French, Schwert and Stambaugh [1987] and Campbell and Hentschel [1992]). The ARCH class of models is now used widely as models of conditional heteroskedasticity in explaining the time series behavior of finance data (see Engle [1982] and Bollerslev [1986]). However, this approach assumes a particular parametric function, which is really just an approximation of the true model and, in addition, assumes a linear symmetric form for the relationship between volatility and the conditioning variables. In this paper, in addition to these two procedures, a nonparametric regression model (a normal kernel regression (KERNEL) model) is employed to estimate the volatility. A nonparametric method has been developed recently to estimate a regression curve without making strong assumptions about the shape of the true regression function (see Silverman [1986]). Moreover, a nonparametric derivative method is used to examine the relationship between the volatility and the level of the stock return.

The analysis in this paper uses daily data on the Japanese *Nikkei 225* from November 1989 to November 1992 which includes a period when the stock market is in a slump. During the whole sample period, there appears to be negative relationships both between the volatility and the past stock return and between the expected stock return and the past stock return, which are interpreted as indirect evidence of a positive relationship between the expected stock return and the volatility. It is also found that the conditional variance exhibits heteroskedasticity, depending on the level of the stock return nonlinearly.

The paper is organized as follows. Section 2 provides details of the three approaches which are used to estimate the volatility of the stock return, and which are extended to examine the relationship between the conditional variance and the level of the stock return. A description of the data and details of the empirical results are presented in section 3. Finally, section 4 contains a brief conclusion.

2 ESTIMATION METHODS

In this section, three econometric methods, an implied volatility regression (IVR) method using the Black Scholes (B-S) model, a parametric estimation method using a GARCH-M model, and a nonparametric estimation method with a KERNEL model, are used to estimate the volatility of stock returns. In addition, the nonparametric methods, which are also used to estimate the partial derivatives, are detailed.

2.1 The IVR Model

The volatility of the stock return can be implied from a particular financial pricing model such as the B-S model. Black and Scholes [1973] derived an option pricing model (the B-S model) which was extremely useful in catalyzing a lot of research on option-like financial instruments. The pricing formulas for the B-S model are

$$c = SN(d_1) - Ke^{-r\tau}N(d_2), \qquad (1)$$

$$p = Ke^{-r\tau}N(-d_2) - SN(-d_1) \qquad (2)$$

where c is the current call option price, p is the current put option price, S is the current price of the underlying stock, r is the risk-free interest rate, K is the exercise price, τ is the time to maturity of the option, and $N(\cdot)$ is the standard normal cumulative density function. The variables d_1 and d_2 are defined as

$$d_1 = \frac{\ln(S/K) + r\tau}{V^{1/2}\sqrt{\tau}} + \frac{V^{1/2}\sqrt{\tau}}{2},$$

$$d_2 = d_1 - V^{1/2}\sqrt{\tau}$$

where $V^{1/2}$ is the standard deviation of the stock's rate of return (volatility). In this model, the volatility is the only unknown parameter. The implied volatility (IV) is estimated using data from several options on the same stock.

In this study, the estimates of IV are computed by applying implied volatility regression. This is a nonlinear least square procedure applied to option prices in the following model (see Latane and Rendleman [1976]),

$$c_{tk} = \Psi_{ctk}(\hat{V}_{ctk}^{1/2}; S, K, r, \tau) + \eta_{ctk}, \qquad (3)$$

$$p_{tk} = \Psi_{ptk}(\hat{V}_{ptk}^{1/2}; S, K, r, \tau) + \eta_{ptk} \qquad (4)$$

where $\Psi_{ctk}(\cdot)$ (or $\Psi_{ptk}(\cdot)$) is the price given in the right handside of (1) (or (2)), η_{ctk} (or η_{ptk}) is a random disturbance, and $\hat{V}_{ctk}^{1/2}$ (or $\hat{V}_{ptk}^{1/2}$) represents the IV for call (c) (or put (p)) options of the kth monthly maturity at time t, respectively. Estimates of $V_{ctk}^{1/2}$ and $V_{ptk}^{1/2}$ are obtained by solving the following minimization problem (see Whaley [1981]):

$$\min_{\hat{V}_{ctk}^{1/2}} G_{ctk} = \sum_{i=1}^{N_{ctk}} (c_{itk} - \Psi_{ictk}(\cdot))^2, \qquad (5)$$

$$\min_{\hat{V}_{ptk}^{1/2}} G_{ptk} = \sum_{j=1}^{N_{ptk}} (p_{jtk} - \Psi_{jptk}(\cdot))^2 \qquad (6)$$

where N_{ctk} (or N_{ptk}) represents the number of call (or put) options, c_{itk} (or p_{jtk}) shows call (or put) options of k months maturity and ith (or jth) exercise price at time t, respectively. The solution to (5) or (6) minimizes the sum of the squared deviations between the observed and calculated option prices.

A cross-sectional estimate of IV is obtained for each option maturity using two methods. First, the initial value of $V_{ctk}^{1/2}$ ($V_{ptk}^{1/2}$) can be estimated using the golden section method (GSS) (see Iwata [1989]), then IV is calculated at the same time using a Gauss search method[1]. Both methods are employed to solve (5) (or (6)), in which the partial derivative of G_{ctk} (or G_{ptk}) with respect to $V_{ctk}^{1/2}$ (or $V_{ptk}^{1/2}$) is set close to zero.

In recent option studies, the IVR model has been the most popular in order to investigate not only the implied volatility of stock returns but also implied stock prices (see Manaster and Rendleman [1982] and Kii [1994]). However, this approach depends on the particular functional form of the specific pricing system adopted, that is, the B-S model. If the B-S model is mis-specified, the volatility estimates may be biased. Indeed, the B-S model makes the strong and quite unrealistic assumptions that the volatilities are nonstochastic and constant

1) The tolerance criterion of the method is 1.0×10^{-6} and 1.0×10^{-8}, respectively.

among each option, each position of option premia, and each maturity.

2.2 The GARCH-M Model

In examining many financial questions, models in the ARCH or GARCH classes have been used successfully to explain the time series properties of some economic variables and are used widely to estimate the parameters of discrete-time models (for example, Nelson [1991] and Chan *et al.* [1992]). These models permit a time varying conditional variance even though the unconditional variance is time invariant, and are used to estimate the volatility parametrically. In this paper, the well known GARCH-M model which is a development of Engle, Lilien and Robins' [1987] ARCH-in-Mean, or ARCH-M model, in which the conditional mean is an explicit function of the conditional variance is used to estimate the stock return volatility.

The discrete-time specification used to estimate the volatility ($V_t^{1/2}$) of the stock return is the following GARCH-M (1,1) process:

$$R_t = \sum_{i=1}^{5} d_{it}\gamma_i + \alpha_1 V_t^{1/2} + \varepsilon_t, \tag{7}$$

$$\varepsilon_t/\Phi_t \sim N(0, V_t),$$

$$V_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 V_{t-1} \tag{8}$$

where R_t is the stock return, d_{it} denotes a day of the week dummy variable taking value 1 when t is the ith day of the week and 0 otherwise, and Φ_t denotes the conditioning variable. In this investigation, ε_{t-1} is chosen as the conditioning variable. In this model, R_t is adjusted to be free from possible day of the week effects and, therefore, the influence of the number of days that have elapsed since the last trading date on changes in the stock prices.

This specification allows the conditional mean to depend on volatility, $V_t^{1/2}$, and if $\alpha_1 = 0$, this model reduces to the basic ARCH or GARCH model. The dependence of the conditional mean function on $V_t^{1/2}$ explains the asymmetric effects of shocks to R_t and captures the volatility feedback effect (see French, Schwert and Stambaugh [1987]), which the ARCH or GARCH model cannot capture because of their constant conditional mean.

In the GARCH-M framework, however, the conditional variance function exhibits a symmetric property with respect to the conditioning variable, ε_{t-1}, whereas the true data generating process may not exhibit this property (see Kogure and Takeuchi [1993]). As well as the ARCH and GARCH models, the GARCH-M model has a restriction that there is a quadratic mapping between V_t and ε_{t-1}, and the predictive feature of V_t induced by the leverage effect cannot be captured using the GARCH-M model.

Although the exponential GARCH (EGARCH) and the quadratic GARCH (QGARCH) model of Engle [1990] and Sentana [1995], respectively take account of this problem and allow V_t to be an asymmetric function of the past data, the linear relationship between the volatility and the stock return is still assumed (see Campbell and Hentschel [1992]). Each of these parametric models is only an approximation of the true model and is known not to capture a complex nonlinearity[2]. If the GARCH-M model is mis-specified so that the true data generating process is too nonlinear to be captured by a parametric model such as a QGARCH model, the estimates of volatility may be biased and the results of a parameter significance test used to examine the correlation of the volatility with the stock return may not be appropriate.

2) The ARCH class of model may neglect nonlinearities in the conditional mean (see Diebold and Nason [1990]). Pagan and Schwert [1990] compare several statistical models, including the ARCH class of models, for the volatility of stock returns and discover an important nonlinearity in the stock returns which cannot be captured by an ARCH type model.

2.3 The KERNEL Method

In estimating the volatility of stock returns, a nonparametric method, which does not need to specify the model parametrically, is also used. This approach will be useful, especially for estimating volatility if the parametric model such as a GARCH-M model is mis-specified and does not adequately explain either an asymmetric feature or a complex nonlinearity in conditional variance. In this investigation, to avoid making strong assumptions about the shape of the true regression function, the normal kernel (KERNEL) model developed by Rosenblatt [1956] is the nonparametric regression model used to estimate the conditional mean, the conditional variance of the stock returns, and their derivatives with respect to the conditioning variables.

Now, in order to avoid a day of the week effect, the stock returns (X_t) is defined as

$$R_t=\sum_{i=1}^{5} d_{it}\gamma_i+X_t \tag{9}$$

where R_t and d_{it} are given by (7). Then, the conditional mean and variance of the stock returns (X_t) given a point (x) can be defined as

$$M_t(x)=E[Y_{1t}|X_{t-1}=x], \tag{10}$$

$$V_t(x)=E[Y_{2t}|X_{t-1}=x]-M_t(x)^2 \tag{11}$$

where $Y_{1t}=X_t$, $Y_{2t}=X_t^2$, and $M_t(x)$ and $V_t(x)$ denote the conditional mean function of x and the conditional variance function of x, respectively. Comparing the GARCH-M framework, this formula permits the conditional mean not to depend on the volatility because the past stock return is assumed to be the only conditioning variable in this framework.

Given X_t is assumed to be strictly stationary and strong mixing to obtain the appropriate asymptotic properties, (10) and (11) can be estimated using

$$\widehat{M}_t(x) = \frac{\sum_{t=2}^{T} Y_{1t} K(w_t)}{\sum_{t=2}^{T} K(w_t)}, \tag{12}$$

$$\widehat{V}_t(x) = \frac{\sum_{t=2}^{T} Y_{2t} K(w_t)}{\sum_{t=2}^{T} K(w_t)} - \left(\frac{\sum_{t=2}^{T} Y_{1t} K(w_t)}{\sum_{t=2}^{T} K(w_t)}\right)^2. \tag{13}$$

$K(\cdot)$ represents the normal kernel function given as

$$K(w_t) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{w_t^2}{2}), \tag{14}$$

$$w_t = (X_t - x)/h \tag{15}$$

where h denotes the band-width. The band-width is chosen to be proportional to $N^{-\frac{1}{4+p}}$ where N denotes the sample size and p is the number of the regressors, so that the mean squared error is minimized (see Silverman [1986]). It is important to note that these kernel estimators will be consistent and asymptotically normal. Hence, an advantage of the nonparametric regression is that the statistical properties of the estimator can be derived with standard techniques.

As the regression function at each x is easily defined as an expectation and density function involving weighted sums in the KERNEL method, the nonparametric regression formula is available for estimating not only the conditional variance but also the partial derivatives of the regression function with respect to the regressors. In particular, the first order derivative is just similar to estimating the regression coefficient and the second order derivative specifies the nonlinear form.

In this paper, a nonparametric derivative procedure with the KERNEL model is applied to estimate the first order derivative of $M_t(x)$ with respect to X_{t-1} and $\partial V_t(x)/\partial X_{t-1}$, in order to investigate two correlations: the negative one, indicating that the conditional variance negatively depends on the past stock returns and the positive one, indicating the conditional mean positively depends on the conditional variance. The first correlation can be directly investigated to estimate $\partial V_t(x)/\partial X_{t-1}$, on the other hand, the second correlation can be indirectly investigated with $\partial M_t(x)/\partial V_t(x)$, using estimates of both $\partial M_t(x)/\partial X_{t-1}$ and $\partial V_t(x)/\partial X_{t-1}$. Moreover, two estimates of the second order derivative of $M_t(x)$ with respect to X_{t-1} and $\partial^2 V_t(x)/\partial X_{t-1}^2$ are used to test for a nonlinearity in the conditional mean and variance of the stock returns.

Using the KERNEL model, the first order derivative of $M_t(x)$ with respect to X_{t-1} and the first order derivative of $V_t(x)$ with respect to X_{t-1} can be estimated, respectively. For $\partial M_t(x)/\partial X_{t-1}$, the estimator $(\widehat{\beta_{mt}^1})$ can be obtained by

$$\widehat{\beta_{mt}^1}(x)=\sum_{t=1}^{T} A Y_{1t},$$

$$A=\left(\frac{w_t\sum_{t=1}^{T}K(w_t)-\sum_{t=1}^{T}w_tK(w_t)}{h\left(\sum_{t=1}^{T}K(w_t)\right)^2}\right)K(w_t) \tag{16}$$

where w_t is again given by (15). For $\partial V_t(x)/\partial X_{t-1}$, the estimator $(\widehat{\beta_{vt}^1})$ is

$$\widehat{\beta_{vt}^1}(x)=\sum_{t=1}^{T}(1-2C)AY_{2t},$$

$$C=\frac{K(w_t)}{\sum_{t=1}^{T}K(w_t)} \tag{17}$$

where A is given by (16).

The second order derivatives of $M_t(x)$ and $V_t(x)$ can also be obtained using the KERNEL model (see McMillan, Ullah and Vinod [1989]). That is, for $\partial^2 M_t(x)/\partial X_{t-1}^2$, the estimator $(\widehat{\beta_{mt}^2})$ can be obtained by

$$\widehat{\beta_{mt}^2}(x)=\sum_{t=1}^{T} B Y_{1t}, \tag{18}$$

and for $\partial^2 V_t(x)/\partial X_{t-1}^2$, the estimator $(\widehat{\beta_{vt}^2})$ is

$$\widehat{\beta_{vt}^2}(x)=\sum_{t=1}^{T}(B-2C-2A^2)Y_{2t},$$

$$B=\left(\frac{w_t^2\sum_{t=1}^{T}K(w_t)-2w_t\sum_{t=1}^{T}w_tK(w_t)-\sum_{t=1}^{T}w_t^2K(w_t)}{h^2(\sum_{t=1}^{T}K(w_t))^2}\right.$$

$$\left.+\frac{2(\sum_{t=1}^{T}w_tK(w_t))^2}{h^2(\sum_{t=1}^{T}K(w_t))^3}\right)K(w_t) \tag{19}$$

where A is again given by (16) and C is given by (17).

In addition, the average derivatives can be calculated as

$$\widehat{\beta_m^1}=\frac{1}{T}\sum_{t=1}^{T}\widehat{\beta_{mt}^1}(x), \tag{20}$$

$$\widehat{\beta_v^1}=\frac{1}{T}\sum_{t=1}^{T}\widehat{\beta_{vt}^1}(x), \tag{21}$$

$$\widehat{\beta_m^2}=\frac{1}{T}\sum_{t=1}^{T}\widehat{\beta_{mt}^2}(x), \tag{22}$$

$$\widehat{\beta_v^2}=\frac{1}{T}\sum_{t=1}^{T}\widehat{\beta_{vt}^2}(x), \tag{23}$$

which are consistent and asymptotically normal, which means that these values can be used in the same way as the estimated coefficients of the parametric regression model (see Rilstone [1991]).

3 EMPIRICAL TESTS

In this section, some empirical results are presented. First, the volatilities of stock returns are estimated using the three methods in the section 2 and are compared with each other. Next, the relationship between the estimated volatility and the level of stock returns is examined by the three econometric tests, corresponding to the three estimation methods, respectively. Third, the possibility that the true conditional mean or variance function of the stock return is nonlinear is investigated.

3.1 Data

The Japanese *Nikkei 225* index is used to define the one-period natural log return on a stock as $R_t \equiv \ln S_t - \ln S_{t-1}$, where S_t is the daily stock price measured at the end of period, t. In estimating the IV, measures of the kth monthly maturity and ith exercise price of *Nikkei 225* index options (c_{itk}, p_{jtk}), and the yield of *CD gensaki* (r_t) are used. All of the data are daily closing values and are taken from the *Nihon Keizai Shinbun*. The data run from 21 November 1989 to 30 November 1992.

The period being investigated includes a dramatic decline of the *Nikkei 225* stock index when the stock prices have been skewed negatively and the daily standard deviations have changed very rapidly. Table 1 presents some summary statistics for both the stock price and the stock return in this sample period.

3.2 Comparisons of the Volatilities Estimates

Table 2 contains the summary statistics for the estimated volatilities of daily stock returns obtained by the three different models : the IVR model for both call and put options in (3) and (4), the GARCH-M model having the parametric form such as (7) and (8), and the KERNEL model which is a standard nonparametric estimation method applied to (12) and (13). This table also includes two graphs plotting the estimates of volatility obtained from both in the GARCH-M and the KERNEL models against the conditioning variable, the past stock return, observed at each point.

Mainly, three results are found in these estimates. First, some interesting features are found in the daily predictions of the volatilities. The standard deviation of the daily volatility estimated by the KERNEL method is the smallest among all the volatility estimates over the whole sample period. In contrast, the mean value for the KERNEL method is greater than that of the GARCH-M estimate and lies between the put IV estimate and the call IV estimate, which is inconsistent with the results in Engle, Kane and Noh [1993]. Moreover, the KERNEL prediction of the daily volatility is more highly correlated with the GARCH-M prediction than the prediction of the IV for call or put options. The movement of the call IV is not similar to the other volatility movements because only the call IV has a negative correlation with the other predictions.

Second, the IVR method provides quite different volatility estimates for call and put options prices, which violates the assumption of the B-S model used in the method. This suggests the possibility of biased estimates.

Third, the figures show that the estimated volatility using the GARCH-M model is symmetric in the stock's rate of return whereas the volatility estimated by the KERNEL method decreases as the past stock return increases. This result is most important and interesting. It suggests that the GARCH-M model may be mis-specified and cannot capture the asymmetric properties of the stock return volatility discussed in the previous sections and that the daily volatility may be negatively correlated with the daily stock return.

3.3 Tests of the Relationship between Returns and Volatilities

Two main approaches, one is the parametric method that is applied to the regressions of the IV or to the GARCH-M model and the other is the nonparametric procedure to estimate and test the nonparametric derivatives with the KERNEL model, are used in the empirical tests that follow.

3.3.1 Regressions with the IV

The volatility feedback effect suggests the higher stock returns volatilities are, the higher expected equilibrium returns are, which shows a positive relationship.

In addition, considering the leverage effect in a Modigliani and Miller (MM) world, conditional variances or volatilities functions may negatively depend on observed past stock returns. In order to investigate these relations between the volatility and the level of stock returns, some simple empirical tests are helpful for the estimated volatilities (the call IV and the put IV) in the IVR model.

In this study, the following simple regression models which are all linear parametric models and are developed to take account of the day of the week are estimated (see Christie [1982] and French, Schwert and Stambaugh [1987]).

$$R_t = \sum_{i=1}^{5} d_{it} a_{1i} + b_1 \widehat{V}_t^{1/2} + \epsilon_{1t}, \tag{24}$$

$$\widehat{V}_t = a_2 + b_2 X_{t-1} + \epsilon_{2t}, \tag{25}$$

$$\ln \widehat{V}_t = a_3 + b_3 \ln S_{t-1} + \epsilon_{3t} \tag{26}$$

where S_t represents the stock prices, $\widehat{V}_t^{1/2}$ denotes the call IV (or the put IV), X_t is defined as in (9), both R_t and d_{it} are given by (7), and $\epsilon_{kt} (k=1, 2, 3)$ is a disturbance. In each equation, if b_k is zero for each $k (k=1, 2, 3)$, then there is no relationship between the volatilities and the stock returns. If b_1 in (24) has a positive value, the expected return is proportional to the volatility, implied by the volatility feedback effect, for example. If b_2 (or b_3) in (25) (or (26)) is negative, the volatility is correlated with the past stock returns (or prices), negatively, which is explained by an effect such as the leverage effect.

Ordinary least squares (OLS) estimates of b_k and the absolute values of the appropriate t statistics for all equations ($k=1, 2, 3$) are presented in Table 3. The null hypothesis that $b_k=0$ cannot be rejected at the 10 percent level in both (24) and (25) for both the call and put IV, which means that there is not a specific relation between the IV for both call and put options and the stock returns in the sample period. Although the t statistics for $b_k (k=1, 2)$ do not reject the null hypothesis of b_k, the values of b_1 in (24) are positive, whereas all values of b_2 in (25) are negative for both call and put IV, which is consistent with the results predicted by the volatility feedback effect and the leverage effect (see Christie [1982], French, Schwert and Stambaugh [1987], and Duffee [1995]). In (26) the estimates of b_3 are negative and their t statistics suggest that the null hypothesis of $b_3=0$ can be rejected at the 10% level for both call IV and put IV. However, these results are inconsistent with the B-S model because the B-S model assumes that 'κ' for call ($=\partial c/\partial V^{1/2}$) or for put ($=\partial p/\partial V^{1/2}$) is positive and '$\delta$' for call ($=\partial c/\partial S$) is positive whereas '$\delta$' for put ($=\partial p/\partial S$) is negative, where $V^{1/2}$, S, c and p represent volatilities, stock prices, and call and put prices, respectively. Given a constant call or put premium, when the data is adequately captured by the B-S model, a negative correlation between the call IV and the stock prices and a positive correlation between the put IV and the stock prices should be obtained.

The results in this investigation are ambiguous and this may be caused by biased estimates. In this examination, there are two problems that should not to be ignored: the 'generated regressor' problem of Pagan [1986] and the model mis-specification problem. When estimates of the volatility obtained from the IVR method assuming that the true data follows the B-S model are used, the estimation procedure suffers from a 'generated regressor' problem. If the pricing model, the B-S model, is not true, estimates obtained by the procedure are affected by mis-specification errors, as suggested above. To examine more explicitly the relationship between the volatility and the stock return, two main approaches with time-series models, the parametric method and the nonparametric procedure, are used in the empirical tests that follows.

3.3.2 GARCH-M Frameworks

As in previous studies, for example, French, Schwert and Stambaugh [1987] and Campbell and Hentschel [1992], in order to examine the relationship between the volatility and the level of the stock returns, a maximum likelihood (ML) estimation procedure is used to estimate the parametric models : (7) and (8) in the GARCH-M framework, which are simultaneously used to estimate the conditional variance, V_t. In particular, the GARCH-M framework which permits the conditional mean to depend on the volatility, $V_t^{1/2}$, is useful to investigate the volatility feedback effect.

Table 4 presents the parameter estimates of (7) and (8), their t statistics, and the maximized value of the log likelihood. In (7), the hypothesis that α_1 is zero cannot be rejected at the 5 percent significance level, which means that the conditional mean does not depend on the volatility and that a clear effect of the volatility feedback to the stock return is not found.

Turning to the conditional variance in (8), the hypothesis that β_1 is zero can be rejected at the 5 percent significance level. This rejects the hypothesis of no correlation between the conditional variance and the past stock returns, even though the earlier evidence suggested the negative relation between the volatility and the realized stock return is weak and uncertain. In addition, the parameter β_2 is not found to be zero at the 5 percent significance level suggesting that the conditional variances are heteroskedastic and are likely to have nonlinear functions.

Both the econometric procedure with the IVR model and the parametric approach using the GARCH-M framework may suffer from a model mis-specification. Suppose the movement of the volatility is inversely proportional to that of the stock return. This is an asymmetric property not captured by the GARCH-M model, so the estimates are biased because of the approximation error. Moreover, when the conditional variance function exhibits a complex nonlinearity, which is not explained by the GARCH-M model, the estimates are also biased. A nonparametric approach that does not need to specify the estimation model parametrically may be useful if the parametric model is mis-specified.

3.3.3 Nonparametric Derivative Procedures

The nonparametric procedure that does not assume a particular financial model nor estimation model is useful not only to estimate the volatility but also to test the relationship between the conditional variance and the level of the stock returns.

At first, in order to investigate the predicted negative correlation, the first order derivative of the conditional variance ($V_t(x)$ in (11)) with respect to the past stock return (X_{t-1} in (9)) can be estimated using (21) for the sample period. The negative relation with the volatility and the level of stock returns cannot be checked obviously in the other two econometric approaches: the regression method with the IV and the GARCH-M method, while the nonparametric procedure which can estimate the relationship directly, using only the data and avoiding model mis-specification errors. A estimate of the partial derivative, $\partial V_t(x)/\partial X_{t-1}$, may provide information about both the dependence of the conditional variance on the level of the stock return and the nature of the heteroskedasticity present. A estimate of the partial derivative, $\partial M_t(x)/\partial X_{t-1}$, may provide information about the dependence of the conditional mean on the level of the stock return. Both estimates of $\partial V_t(x)/\partial X_{t-1}$ and $\partial M_t(x)/\partial X_{t-1}$ may indicate the correlation between $M_t(x)$ and $V_t(x)$ because $\frac{\partial M_t(x)/\partial X_{t-1}}{\partial V_t(x)/\partial X_{t-1}} = \partial M_t(x)/\partial V_t(x)$, which may be used in order to investigate the positive correlation of the volatility with the expected stock return.

Table 5 includes estimates of the average first order derivatives using the KERNEL model. The value of $\partial V(X_t)/\partial X_{t-1}$ is negative and the hypothesis that $\partial V(X_t)/\partial X_{t-1}=0$ can be rejected at the 5% level. This result means that the volatility is heteroskedastic and depends negatively on the past return in the

Japanese stock market. The value of $\partial M(X_t)/\partial X_{t-1}$ is also negative and the hypothesis that $\partial M(X_t)/\partial X_{t-1}=0$ can be rejected at the 5% level. The two results indirectly show the positive correlation between the expected return and the volatility, which induces the effect of the volatility feedback in the Japanese stock market.

In addition, to examine how the stock return is correlated with the volatility and the nature of the nonlinearity in the conditional variance, the average second order derivatives of the nonparametric regression are estimated. The second order derivative of $V_t(x)$ with respect to X_{t-1} can be estimated using (23). Estimates of the average second order derivatives in (23) using the KERNEL function are also presented in Table 5 for the sample period. The null hypothesis that $\partial^2 V(X_t)/\partial X_{t-1}^2=0$ can be rejected at the 5% level. This result suggests that a complex nonlinearity may exist in the conditional variance of the Japanese stock return which is evidence of the mis-specification in the simple regression and GARCH-M models, and that the two other parametric approaches may be inadequate to estimate the volatility and to test the correlation with the stock return.

4 CONCLUSION

In this paper, the relationships between the stock returns and their volatilities estimated by the several methods are investigated using data for Japan.

This paper has two essential contributions. In the first, three different type approaches: the IVR method assuming a particular pricing model, the GARCH-M method which specifies a parametric model as an approximation, and the nonparametric method which does not make strong assumptions about the shape of the true regression function, are used to estimate the conditional variance and then the volatility of the stock returns. Secondly, the nonparametric method is used to directly test the relationship between the estimated volatility and the stock returns. None of the earlier studies on the subject have used a nonparametric procedure to not only estimate the volatility but also to test this relationship.

There are two important conclusions. First, both a negative correlation between the volatility and the level of the realized excess holding return and a positive correlation of the volatility with the expected stock return can be found in the Japanese stock market. This finding can give an evidence to the previous studies suggesting the relationships between the volatilities and the stock returns in the U.S. market, which is usually explained by the leverage effect and the volatility feedback effect (for example, Christie [1982] and French, Schwert and Stambaugh [1987]).

Second, some time-series parametric models such as a GARCH-M model that are offen used in this type of investigation are found to be inadequate. In contrast to the nonparametric model, the parametric models cannot explain an asymmetric property such as the negative correlation of the volatility conditioned on the realized stock returns. Moreover, a nonlinearity can be found in the conditional variance of the Japanese stock returns. The nonlinearity cannot be clearly captured by the usual parametric models, which indicates that the parametric models may not explain the actual market behaviour and should not be used to test in examining the data.

Although there are some problems with the derivative estimates[3], closed-form solutions for stochastic diffusion models of stock prices (or stock returns) including what properties, which the most commonly used models in

3) For example, the possible existence of an estimation bias remains in estimating first or second partial derivatives with the KERNEL model and is left for future research (see Takeuchi and Ohya [1995]).

the valuation of contingent claims ignore, need to be derived and used to examine the predicted correlations between the stock returns and the volatilities, more directly.

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Table 1: Summary Statistics

Variables	N	Mean	Std.Dev.
S_t	743	24621.258	6032.349
R_t	743	−0.0000282	0.0177

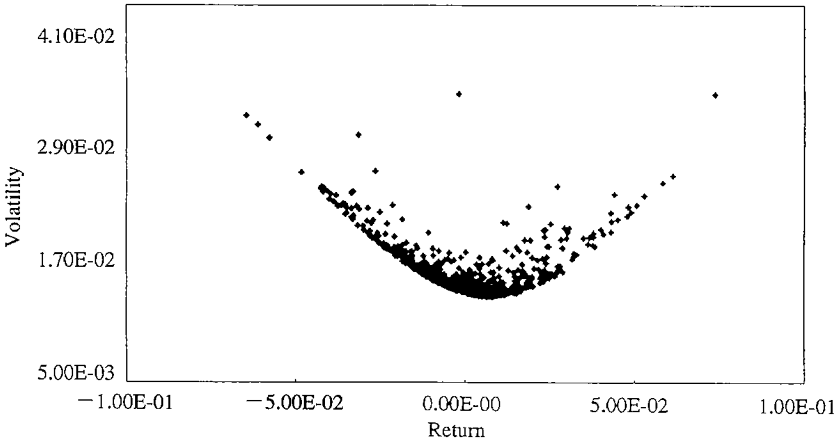
Note: The daily data on the Japanese *Nikkei 225* index (S_t) run from 22 November 1989 through to 30 November 1992. The variable R_t represents the stock return defined as $R_t \equiv \ln S_t - \ln S_{t-1}$. N denotes the number of observations, and Mean and Std.Dev. are the average and the standard deviation of the variables, respectively

Table 2: Estimates of the Volatility

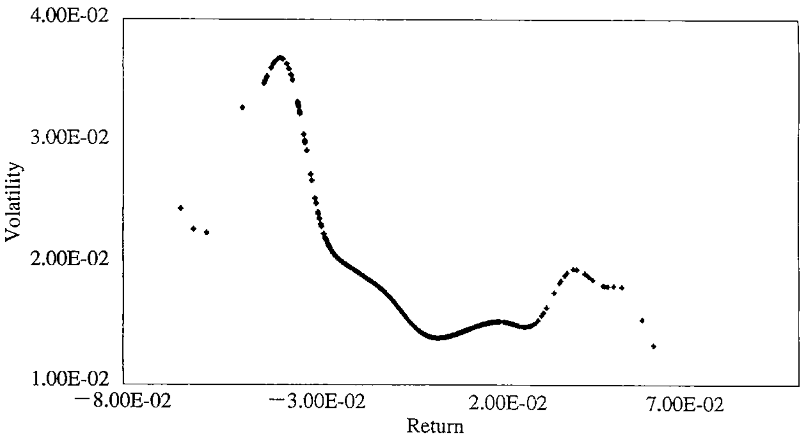
	Mean	Std.Dev.	Min	Max
IV (Call)	0.0201	0.00963	0.000638	0.0553
IV (Put)	0.0154	0.00769	0.000299	0.0514
KERNEL	0.0167	0.00442	0.000136	0.0368
GARCH-M	0.0129	0.0122	8.68 D-6	0.124

	KERNEL	GARCH-M	IV (Call)	IV (Put)
KERNEL	1.000	—	—	—
GARCH-M	0.333	1.000	—	—
IV (Call)	−0.0504	−0.111	1.000	—
IV (Put)	0.0567	0.0686	0.00818	1.000

Note: The volatilities of the rate of returns of the *Nikkei 225* index are computed using the nonparametric method (KERNEL), the GARCH-M method (GARCH-M), and the IVR method for both the call *Nikkei 225* option (IV (Call)) and the put *Nikkei 225* option (IV (Put)). Mean is the average, Std.Dev. is the standard deviation, Min is the minimum value, and Max is the maximum value of these estimates, respectively. The matrix of correlations among the estimates of the volatility is also shown below this table. This table includes two figures named 'Volatility v.s. Return' below, which plot the volatilities (Volatility) estimated with the GARCH-M model (GARCH-M) and the KERNEL model (KERNEL) versus the past stock rate of returns (Return), respectively.



Valatilty v.s. Return
(GARCH-M)



Valatilty v.s. Return
(KERNEL)

Table 3: OLS Estimates with the IV

	b_k
Call (k=1)	0.348 (0.486)
Put (k=1)	0.0688 (0.768)
Call (k=2)	−0.000130 (0.114)
Put (k=2)	−0.000591 (0.865)
Call (k=3)	−0.321 (1.690)*
Put (k=3)	−2.499 (12.801)*

Note: If $k=1$ (or $k=2$, $k=3$), the OLS model is (24) (or (25), (26)) including b_k for the call (Call) and put (Put) IV, respectively. The absolute value of the t statistics of the parameters are reported in parentheses. The 10% critical values for the t statistics are 1.64 and a superscript * indicates that the parameter is statistically different from zero at the 10% level.

Table 4: ML Estimates of the GARCH-M Model

α_1	β_0	β_1	β_2
0.000311 (0.0814)	8.16-3 (3.541)*	0.0892 (4.584)*	0.563 (5.336)*

Note: The maximized value of the log likelihood function is 1216.6. The absolute values of the t statistics of the parameters are reported in parentheses. The 5% critical values for the t statistics are 1.96 and a superscript * indicates that the parameter is statistically different from zero at the 5% level.

Table 5: Derivatives of the Nonparametric Regressions

	estimate	χ^2
$\partial M(X_t)/\partial X_{t-1}$	−0.0772	9.968*
$\partial^2 M(X_t)/\partial X_{t-1}^2$	−4.277	0.401
$\partial V(X_t)/\partial X_{t-1}$	−0.00732	35.654*
$\partial^2 V(X_t)/\partial X_{t-1}^2$	0.326	4.174*

Note: The χ^2 value is a test of the null hypothesis that the mean of the partial derivative is equal to 0. The 5% critical value for the $\chi^2(1)$ statistics is 3.84 and a superscript * indicates that the null hypothesis that the derivative is zero can be rejected at the 5 percent significance level.