【論 説】

Money and Growth with Public Capital *

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I Introduction

The effects of monetary policy on capital accumulation and inflation have been examined in considerable detail. To a large extent, this has been done with the aid of neoclassical money-growth models. Most notable are the studies of Tobin (1955, 1965), Sidrauski (1967), Johnson (1967, Chapter 4), Levhari and Patinkin (1968), and Mundell (1971, Chapter 5). This paper extends these earlier studies by introducing publicly-supplied capital into the analysis. With this modification, it becomes possible to demonstrate that inflation and the accumulation of private capital will also be sensitive to policies governing the accumulation of public capital. This occurs fundamentally because such policies affect the disposable income of the private sector and the rates of return on privately-supplied factors of production.[1]

Following Uzawa (1971 and 1972) and Oakland (1972), a distinction is drawn between the actual degree of utilization of the services of public

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1) As a first approximation, the direct effects of public goods on the demand function of the household sector are ignored.

capital and the capacity level of those services.[2] The former is subjectively determined by the private sector although it is responsive to the supply conditions of such services as determined by the government. The latter, on the other hand, is a datum to the private sector since it is exogenously constrained by the stock of public capital.

The discussion to follow assumes that the markets for commodities, equities, labour and money are competitive. There is, however, no effective market for publicly-supplied factors, and the utilization of these services produces external diseconomies. The structure of the model and the equilibrium of the economy are described in Section II. The basic components of the model are the specification of the production technology of the aggregate commodity, the consumption function and the portfolio of assets. Sections III and IV deal with impact and long-run effects of changes in the rate of growth of money and in the level of public capital. Section V summarizes main conclusions of the analysis and some qualifications of the model are briefly discussed.

II Model and Equilibrium

The components of the model are presented in terms of the following hypotheses connecting the variables of the system:

i) The labour force and the private capital are fully employed. Furthermore, the labour force, L, grows at a given positive rate, n.

ii) Net accumulation of the real stock of public capital, Z, comes about through (a) depletion which is linearly related to X, the amount of

2) For example, the capacity level of the services of a publicly-supplied road refers to the maximum possible intensity of the road services which the private sector could use in a given period. This is different from the rate of utilization actually chosen by the private sector.

the services of public captial used by the private firms, and (b) new government outlay, G. In terms of redefined units, the dynamics of z is given by

$$\dot{z}=-\alpha x+g-nz,\ \alpha>0;\ z=Z/L,\ g=G/L, \tag{1}$$

where α is constant.

iii) The government continuously aims at preserving a constant growth rate, μ, of the nominal stock of money, and a constant level, θ, of real stock of public capital per worker. That is,

$$\dot{M}/M=\mu, \text{ and } z=\theta, \tag{2}$$

where M is the nominal aggregate stock of money. The second condition in (2) implies that the government undertakes the obligation to supply a certain level of public capital per head.[3] Note that if the government follows this rule, then $\dot{Z}/Z$ will equal n at all times.

iv) Government expenditure equals T, net transfer payments to the private sector in a form of money, plus G, public investment. $T+G$ is financed by money creation:

$$T+G=\mu(M/p), \tag{3}$$

where p is the money price of the aggregate good. In sum, the government chooses a pair of values of μ and θ, and thereby adjusts M and G in each period so as to maintain (2) under the budget constraint, (3).

v) The output of the aggregate good, Y, depends on two privately-supplied inputs (private capital, K, and labour), the capacity level of public capital, and the use rate of the services of public capital. Assuming a linear homogeneous production function, it is specified

3) As we shall discuss in Section V, this assumption is necessary for attaining the long-run growth equilibrium.

by

$$y=f(k,v)\ ;\ y=Y/L,\ k=K/L, \tag{4}$$

where

$$v=j(x/z)x,\ j(1)=0,\ j'(.)<0,\ j''(.)<0.^{4)} \tag{5}$$

In (5), the efficiency coefficient, $j(.)$, of using a unit of the services of public capital is a decreasing function of x/z (for example, the degree of congestion). We assume further that

$$f_i(.)>0,\ f_{ii}(.)<0,\ f_{ij}>0,\ \text{for } i,j=k,v, \text{ and } i\neq j. \tag{6}$$

From (5) and (6), the marginal productivities of private capital and capacity public capital are positive and decreasing. The marginal productivity of the use rate of the services of public capital can be traced to two sources, namely, an internal economy and an external diseconomy. This marginal productivity can be positive or negative:

$$f_x=f_v(.)j(.)+f_v(.)j'(.)(x/z)\gtreqless 0. \tag{7}$$

However, it is always decreasing because

$$f_{xx}=f_{vv}(.)[v_x(.)]^2+f_v\ (.)v_{xx}(.)<0.$$

In sum, using (4) ~ (6), we find the following:

$$\left.\begin{array}{l} f_k>0,\ f_z>0,\ f_x\gtreqless 0, \\ f_{kz}>0,\ f_{xk}\gtreqless 0,\ f_{xz}\gtreqless 0, \\ f_{ii}<0,\ \text{for } i=k,x,z. \end{array}\right\} \tag{8}$$

vi) Competitive markets for privately-supplied factors are assumed. There is no effective market for the services of public capital and the utilization of these services produce external diseconomies. We postulate, *a la* Uzawa, that each member of the private sector will

4) The author is indebted to suggestions provided by T. Miyao concerning a number of issues related to this specification.

utilize the services of public capital so as to maximize his own benefit. It follows that the use rate of a publicly-supplied factor, like the demand for any input, can be derived from the profit-maximizing behaviour of firms. Under competitive conditions, it is assumed that if firms can adjust their allocation of private factors and the use rate of public capital without delay they will maximize their profit in each period. Thus

$$r=f_k(k,x\,;\,z), \qquad (9\text{–}1)$$

$$\tau=f_v(k,x\,;\,z)j(x/z), \qquad (9\text{–}2)$$

$$w=f(.)-[kf_k(.)+xf_x(.)+zf_z(.)], \qquad (9\text{–}3)$$

where r and w are the real rental and the real wage rate, respectively. Equation (9–2) implies that private firms disregard the externality and equate the *private* marginal productivity of the services of public capital employed (i.e., $f_v j$ in (7), not f_x itself) to its private factor cost, τ. In the above, r and w are determined by competitive markets whereas τ and z are set by the government.

vii) For the purposes of the present discussion, one may envision two extreme rules relating to the user's cost of public capital; first, no direct fees are collected from users, and second, a unit fee is set equal to the external production diseconomy, i.e., the marginal social cost of production. The second rule might be adopted by a government attempting to force firms to internalize the production externality. It could then distribute the revenues derived to consmuers.

viii) When no direct cost is charged, $j(x/z)$ must, from (9–2) and (6), equal zero. That is, the use rate of public capital is uniquely determined by the existing stock of public capital supplied by the government. By redefining units, we can specify that $j(x/z)=0$ when

$x=z$, as assumed in (5). This result is identical to the traditional one [*a la* Samuelson (1954)].

ix) When the government charges a unit cost for the use of public capital equal to its marginal social cost of production $[f_v(.)j'(.)\times(x/z)]$, the latter will be equated to the private marginal productivity of the services $[f_v(.)j(.)]$ by profit maximizing firms. As a result, (9–2), using (7), becomes

$$f_x(k,x\,;\,z)=f_v(.)[j(.)+j'(.)(x/z)]=0. \qquad (10)$$

Under this rule of pricing, aggregate output is divided into three parts: (a) the rental income accruing to the owners of private capital, (b) the wage income of workers, and (c) the government revenues arising from the fees charged for the services of public capital.[5] However, since the government, by assumption, redistributes its revenues to the private sector, y equals gross private income per person. It is possible to derive a number of interesting results relating to this rule of pricing. First, the use rate, x, of the services of public capital is again uniquely determined by the existing stock of public capital because (10) depends only on x/z. Second, the use rate of public capital is lower than the one prevailing in the case where the user's cost is zero. This occurs because the value of $j(.)$ in (10) is positive while that in the case of $\tau=0$, it is zero, and because $j(.)$ is a decreasing function of x/z. These two results can be summarized by writing

$$x=\beta z\,;\; 0<\beta<1, \qquad (11)$$

5) From (4) and (5), $f_z z=f_v j'(x/z)x$ which equals the total fee collected by the government. By using (9–1) and (10), (9–3) will give
$y=rk+w+[-f_n j'(x/z)]x.$

where β is constant. Third, given k, an increase in z leads to an increase in per capita output. This result is obvious since

$v=j(\beta)\beta z$ in (4) will increase with z.

x) In the following sections, it will be assumed that the government sets a fee for the services of public capital equal to (a) the associated marginal external production diseconomy, or (b) zero. Since the analysis in the model employing (b) is a special case of the one embodying (a), only the latter will be considered in detail. However, the assumption (b) will be discussed briefly in Section IV.

xi) Once firms determine x in each period from their profit maximizing behaviour, the government decides how much to invest in public capital in order to maintain $z=\theta$. Thus, from (1), (2), and (11), the per capita value of government investment is given by

$$g=(\alpha\beta+n)\theta. \tag{12}$$

xii) The real disposable income of the private sector, Y^d, equals gross income plus the government transfer net of taxes, i.e.,

$$Y^d=Y+T-\pi(M/p)$$

where π is the anticipated rate of inflation and the last term signifies an inflation tax. By using (3), per capita disposable income can be expressed as

$$y^d=y+(\mu-\pi)m-g\,;\ y^d=Y^d/L,\ m=M/(pL), \tag{13}$$

where $[g+(\pi-\mu)m]$ is the net tax burden.

xiii) Each individual consumes a constant ratio, $1-s$, of his disposable income and he makes his portfolio decisions such that the ratio of real cash balances to real capital (equity) depends only on the nominal rate of interest. That is, the behaviour of the household sector is described, in its per capita form, by

$$sy^d=(\mu-\pi)m+(\dot{k}+nk), \text{ and} \tag{14}$$

$$m=\lambda k, \tag{15}$$

where

$$\lambda=\lambda(r+\pi)>0, \text{ and } \lambda'(.)<0. \tag{16}$$

xiv) People adjust their anticipated rate of inflation by an adaptive process, i.e.,

$$\dot{\pi}=\gamma[(\dot{p}/p)-\pi], \ \gamma>0, \tag{17}$$

in which γ, an expectations coefficient, is constant.

The abstract economy which we have described can be reduced to a pair of autonomous differential equations in k and π, i.e.,

$$\phi(k, \pi: \mu, \theta)=\dot{k}/k=(s/k)[f(.)-g]-[(1-s)(\mu-\pi)\lambda(.)+n], \tag{18}$$

$$\psi(k, \pi; \mu, \theta)=\varepsilon\dot{\pi}=\mu-\pi-n-\eta(\dot{k}/k), \tag{19}$$

in which the symbols ε and η are defined as

$$\varepsilon=(1/\gamma)+(\lambda'/\lambda), \text{ and}$$

$$\eta=\partial lnm/\partial lnk=1+(\lambda'/\lambda)k(dr/dk)>1. \tag{20}$$

Equation (18) can be obtained by substituting (4), (13), (15) into (14), and equations (19) is derived from (15) and (17).

To facilitate interpretation, equation (18) can be rewritten as

$$\dot{k}=sy^d-(\mu-\pi)m-nk, \tag{21}$$

which is equivalent to (14). That is, total saving minus the accumulation of real cash balances minus the capital required for the new generation equals the net additional to the stock of private capital. The meaning of the right hand side of (19) can be seen more clearly, if we multiply it by m, thereby obtaining

$$-h=(\mu-\pi)m-(\eta\lambda\dot{k}+nm), \tag{22}$$

where $-h$ represents dishoarding. This shows that dishoarding equals the increase in real cash balances minus the sum of the increase in the demand for money due to private capital accumulation and the money required for the new generation.

The long-run growth equilibrium is attained when $\dot{k}=\dot{\pi}=0$. Hence, in the stationary state we find that

$$\pi^*=(\dot{p}/p)^*=\mu-n, \tag{23}$$

where an asterisk denotes the stationary state level. By substituting (23) into (21), we also obtain

$$s(y^d)^*=n(m^*+k^*),$$

i.e., in the long-run growth equilibrium, saving exactly covers the assets required for the new generation. Furthermore, it can be shown that the long-run growth equilibrium will be locally stable if and only if

$$\phi_{k^*}<0, \text{ and } \varepsilon>0,^{6)} \tag{24}$$

since

$$\phi_{\pi^*}=(1-s)\lambda^*>0,\ \psi_{\pi^*}=-(1+\eta\phi_{\pi^*})<0,$$

and

$$\psi_{k^*}=-\eta\phi_{k^*}.$$

This stability condition is equivalent to say that, in π-k plane, the $(\dot{\pi}=0)$ schedule is steeper than $(\dot{k}/k=0)$ schedule and both are positively sloped.

III Public Capital and Monetary Policy

In this section, we study how the economy adjusts in both the long and short runs to changes in public capital and monetary policies, under the assumption that the stability conditions (24) are satisfied. We begin

6) $\phi_k^*=(s/k^2)\ [f_k k-(f-g)]-(1-s)n(\lambda^*)'(d\gamma^*/dk^*)$. In Sidrauski (1967), this is always negative for $g=0$ and $f-f_k k=w>0$. Therefore, his model is locally stable if $\varepsilon>0$.

with the long-run effects of an increase in the government's target level of public capital per person θ.

By setting (18) and (19) equal to zero and differentiating, we obtain the following relation which must hold in long-run equilibrium:

$$\begin{bmatrix}\phi_k & \phi_\pi \\ \psi_k & \psi_\pi\end{bmatrix}\begin{bmatrix}dk^* \\ d\pi^*\end{bmatrix}=-\begin{bmatrix}\phi_\theta \\ \psi_\theta\end{bmatrix}d\theta-\begin{bmatrix}\phi_\mu \\ \psi_\mu\end{bmatrix}d\mu. \tag{25}$$

The long-run effect of an increase in θ can be found by solving (25) in terms of θ. This calculation yields

$$\partial k^*/\partial\theta=-(\phi_\theta/\phi_k), \tag{26}$$

$$\partial\pi^*/\partial\theta=0. \tag{27}$$

Equation (27) implies that the long-run equilibrium rate of inflation is unaffected by changes in θ. This is equivalent to the result obtained in (23). Assuming stability, equation (26) implies that an increase in θ can cause the stationary state level of k^* to increase or decrease, depending on the sign of ϕ_θ. The quantity ϕ_θ can be written as

$$\begin{aligned}\phi_\theta&=(s/k)[(\partial y/\partial\theta)-(\partial g/\partial\theta)]-(1-s)(\mu-\pi)\lambda'(\partial r/\partial\theta)\\&=(s/k)[f_z-(\alpha\beta+n)+\xi k],\end{aligned} \tag{28}$$

where

$$\xi=[(1-s)/s]n\lambda'(\partial r/\partial\theta)=-[(1-s)/s]n\lambda' f_{kz}>0$$

[on account of (9–1) and (11)].

For the sake of clarity of exposition we shall assume that

$$f_{ij}=\text{constant for all } i,\ j, \text{ and } \lambda'=\text{constant}.$$

This is effectively the same as assuming that ξ is constant. In order to obtain a criterion for determining the sign of ϕ_θ around the long-run growth equilibrium, let us define $\hat{k}$ as a value of k which makes $\phi_\theta=0$, i.e., for which

$$f_z(\hat{k},\ \beta\theta,\ \theta)+\xi\hat{k}=\alpha\beta+n. \tag{29}$$

Now, since the left hand side of (29), for any given θ, is a monotonously increasing function of k, we know that

$$\phi_\theta \gtreqless 0 \quad \text{when} \quad k \gtreqless \hat{k}. \tag{30}$$

It follows that, when θ is raised, the $(\dot{k}/k=0)$ schedule will rotate counter-clockwise around the point of intersection of $(\dot{k}/k=0)$ and $(k=\hat{k})$. The same thing happens with a rotation of the $(\dot{\pi}=0)$ schedule. In general, two alternative cases can arise with respect to the shifting of $(\dot{k}/k=0)$ and $(\dot{\pi}=0)$ schedules, depending on the relative size of $\hat{k}$ and $k_0{}^*$ (the initial stationary state value of k). These are shown in Figure 1 and Figure 2 respectively.

If $k_0{}^*$ is greater than $\hat{k}$ as in Figure 1, ϕ_θ will be positive around the initial long-run equilibrium, Q. From (26), the new long-run equilibrium

Figure 1

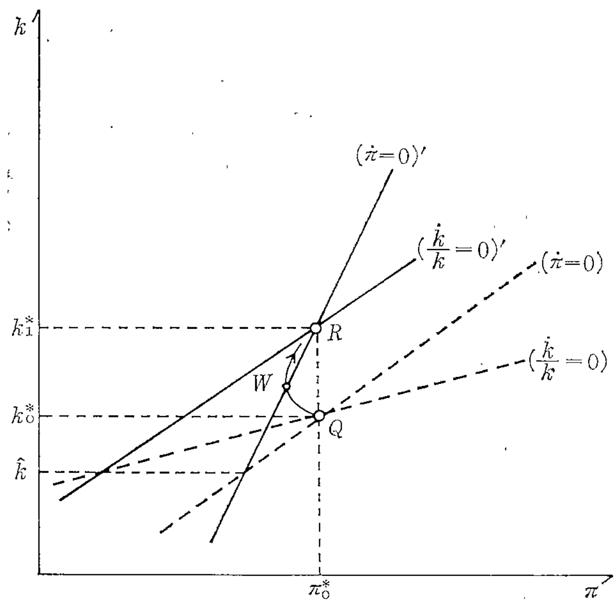
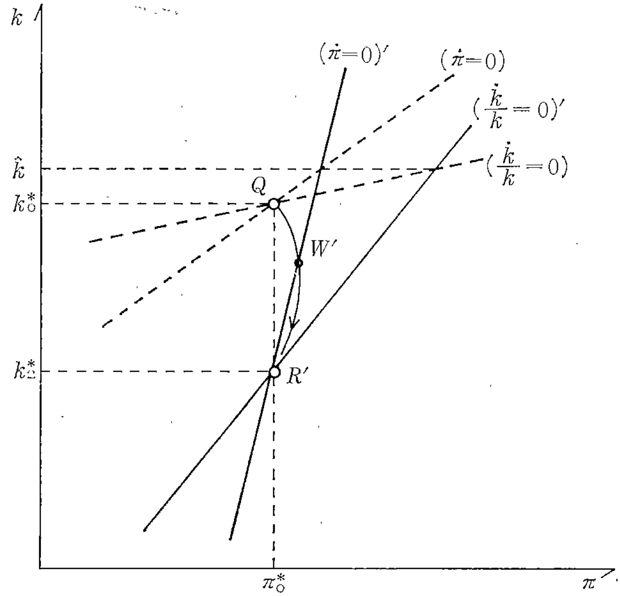


Figure 2



growth path will result in a higher capital-labour ratio in the private sector (e.g., k_1* in Figure 1). If k_0* is less than $\hat{k}$ as in Figure 2, private capital intensity declines. However, the long-run effects on the real cash balances and real disposable income per person are not symmetrical: when k_0* is greater than $\hat{k}$, the real rate of interest may increase or decrease in the new stationary state, since an increment of the real rate of interest due to an increment in θ may be offset by private capital accumulation. Hence, the desired money-equity ratio may increase or decrease, although equity holdings must increase. When k_0* is less than $\hat{k}$, both the long-run equilibrium value of real cash balances and real disposable income will decline.[7]

7) We may verify that ↗

Next, let us examine the short-run impacts on private capital accumulation and inflation of a unit increse in θ. Because the argument is symmetrical, only the case where ϕ_θ is positive around Q will be discussed. With an unchanged k and π, the immediate effects of an increase in θ are (a) an increase in real output, (b) an increase in public investment expenditure, and (c) as can be seen from (9–1), a rise in the real rate of interest. The latter will, in turn, cause λ to decrease. As a result, the increase in real output available for private capital accumulation will be an excess of the increase in total saving over the increase in hoarding. This can be seen from

$$\begin{aligned}\Delta k &= s(\Delta y - \Delta g - \lambda_0' k_0 \Delta r)\\ &\quad -(\mu - \pi_0)\lambda_0' k_0 \Delta r (>0 \quad \text{if } \phi_\theta > 0),\end{aligned} \tag{31}$$

which is derived from (21). A subscript (0) denotes an initial equilibrium value and a delta (Δ) implies a change in the initial moment. In (31), $(\Delta y - \Delta g - \lambda_0' k_0 \Delta r)$ represents a change in disposable income and the last term is the change in hoarding due to a rise in the real interest rate. The short-run effects of the increase in θ on inflation can be analyzed with the aid of (22). An increase in θ initially affects the real rate of interest and private capital accumulation, and thereby real cash balances. The change in dishoarding is given by

$$\Delta(-h) = (\mu - \pi_0 - n)\Delta m - \eta \dot{k}_0 \Delta \lambda - \eta \lambda_0 \Delta k. \tag{32}$$

Since the first two terms are zero around the initial long-run equilibrium, dishoarding will decrease from zero by the amount $\eta\lambda_0 \Delta k$, and hence the actual rate of inflation will fall. Assuming that $\varepsilon > 0$, individuals will

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↘ $\partial m^*/\partial\theta = \lambda^*\eta(\partial k^*/\partial\theta) + (\lambda^*)'(\beta f_{kx} + f_{kz})k^*$, and

$\partial(y^d)^*/\partial\theta = (\lambda^*\eta + r^*)\lambda(\partial k^*/\partial\theta) + f_{z^*} - (\alpha\beta + n) + (\lambda^*)'\lambda(\beta f_{kx} + f_{kz})k^*$.

Hence, $\phi_\theta < 0$ is a sufficient condition for having both expressing being negative.

lower their anticipated rate of inflation. The preceding can be confirmed from (19) which yields

$$\partial\dot{\pi}/\partial\theta = -(\eta/\varepsilon)\phi_0.$$

As time goes on, the economy will accumulate real private capital and consume more. However, the increment in the supply of the good available for private capital accumulation will decrease because the marginal productivity of private capital declines as k increases. At the same time, hoarding will decrease to zero, and it will become negative after the point W in Figure 1.

An increase in the rate of growth of the money supply generates a higher rate of inflation; however, the real output and capital intensity of the private sector initially declines but eventually rises. These results are consistent with the well-known proposition presented by Sidrauski (1967). The rationale within the context of the present study is as follows: Initially, disposable income will increase by the same amount as the increase in the transfer payment. This stimulates consumption and, given that real output and public investment are unchanged, the rate of private capital accumulation declines. At the same time, people start to dishoard because (a) the actual quantity of money has been raised by the government and (b) the demand for money declines because of private capital decumulation. Thus a higher inflation will take place. As time goes on, real private capital per head will decrease with the result that the real rate of interest will increase. The higher rate of inflation and the increase in the real interest rate will make people realize that equity holdings are more efficient, and this will tend to increase private capital accumulation. The long-run effects of an increase in μ is easily derived by solving (25) with respect to μ, i.e.,

$$\partial k^*/\partial\mu = -(\phi_\mu + \phi_\pi)/(\phi_k) = n(1-s)\lambda'/(\phi_k) > 0,$$

and $$\partial\pi^*/\partial\mu = 1.\text{[8]} \qquad (33)$$

IV No Direct Cost of The Use of Public Capital

So far, we have discussed the effects of public capital and monetary policies under the assumption that the government collects a fee directly from the private users of the services of public capital equal to its marginal social cost in production. In contrast, this section deals with the case in which the government sets the fee equal to zero. In this case, τ in (9–2) is zero, with the result that $j(x/z)$ must always be zero as well. In other words, firms will employ the services of public capital up to their saturation level so that (10), (11), and (12) must be replaced by

$$j(x/z) = 0, \qquad (10)'$$

$$x = z = \theta, \qquad (11)'$$

$$g = (\alpha + n)\theta. \qquad (12)'$$

By using Euler's theorem, it can be shown that total output is exhausted by the owners of private factors: hence, y will equal gross per capita income. Both the real rate of return on the private capital and per capita output are not affected, given k, by a change in θ. This occurs because $v = j(1)\theta$ is always zero in (9–1) and (4), with the result that (28) becomes

$$\phi_\theta = -(s/k)(\partial g/\partial\theta) = -(s/k)(\alpha + n) < 0. \qquad (28)'$$

When θ is raised, both the $(\dot{\pi} = 0)$ and $(\dot{k}/k = 0)$ lines always shift downwards and the transition path to the new long-run growth equilibrium is

8) For more detailed discussions on the effects monetary policies, see, for example, Sidrauski (1967), Levhari and Patinkin (1968), Mundell (1971), Foley and Sidrauski (1971), and Dornbusch and Frenkel (1973).

uniquely shown by $QW'R'$ in Figure 2.

The preceding result can be interpreted as follows: An increase in public capital per head by an amount $\Delta\theta$ initially affects neither output per capita nor the rate of return on private capital. However, per capita real disposable income decreases by $(\alpha+n)\Delta\theta$, i.e., by an amount equal to the increase in the tax burden associated with the increase in public investment. Consumption demand decreases by $(1-s)(\alpha+n)\Delta\theta$, but there is no change in hoarding because π, k, and r are unchanged. The decrease in real output available for private capital accumulation will equal the increase in public investment plus the increase in private consumption, i.e., it will equal $s(\alpha+n)\Delta\theta$. This is seen from

$$\Delta k = s(-\Delta g) < 0, \qquad (31)'$$

which is derived form (21). The private capital decumulation is followed by a decrease in the demand for money due to both the lower level of private capital holding and hence a higher interest rate. This leads people to start dishoard with a result that the higher inflation will take place. Assuming that $\varepsilon > 0$, people will adjust their anticipated rate of inflation to a higher level. The subsequent adjustment is analogous to that developed in Section III. In the new long-run growth equilibrium, the rate of inflation returns to its original level, while private capital intensity and per capita output are lower. Per capita real cash balances also decrease due to their higher opportunity cost (i.e., $\Delta r^* > 0$, $\Delta\pi^* = 0$) and to the lower private capital stock per capita.

As for the effects of monetary policy, it can easily be seen that the analysis will be identical to those developed in Section III.[9]

9) Note here that when $\tau = 0$,

$$\phi_k^* = (s/k^{*2})(g^* - w^*) - (1-s)n(\lambda^*)'(dr^*/dk^*) \quad \nearrow$$

V Conclusion

So far, we have assumed that the government sets the growth rate of aggregate public capital equal to the natural growth rate of the economy. This assumption is necessary for the economy to attain the steady growth equilibrium.

This remak can be analyzed as follows: suppose that the volume of public capital is allowed to grow at a rate lower than the natural rate of growth at all times. The stock of public capital per effective worker will eventually approach zero, resulting in a situation in which the level of production becomes unacceptably low. If, on the other hand, the government lets the volume of public capital grow at a rate higher than the natural rate of growth at all times, the volume of public capital per worker will tend to grow indefinitely large, thus calling for an ever increasing tax burden. But real output cannot grow at the same rate because of decreasing marginal productivity and, consequently, consumption declines indefinitely. Thus, it is clear that the government cannot maintain a positive or negative discrepancy between the natural growth rate and the growth rate of the public capital over an extended period of time. In other words, the government could, in principle, control the stock of public capital in an arbitrary manner in the short run but not in the long run.

We have shown that most of the important theorems of the available money-growth literature remain valid when account is taken of public capital. Specifically, when the rate of monetary expansion is raised, the

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↘ which is assumed to be negative for the stability.

capital intensity in the private sector will fall initially, because of the higher level of disposable income and the resulting higher rate of consumption, but it will tend to rise eventually. The rate of inflation increases during the transition period and in long-run equilibrium it equals the difference of the rate of monetary expansion and the long-run rate of economic growth.

The introduction of public capital has enabled us to extend these earlier results by allowing for changes in public captial policies. When the government raises the level of public capital per head, real output, the real rate of return on the private capital, the tax burden, hence the real disposable income and the portfolio structure are immediately affected. If, in the initial moment, the increase in total saving induced by the change in disposable income is less than the increase in hoarding due to the higher interest rate, (a) capital intensity in the private sector, (b) real output, (c) real disposable income, and (d) real cash balances will all decrease along the new long-run growth path when the government raises the level of public capital per head. In this event, the rate of inflation will increase initially along its disequilibrium growth path but it will then decline to its original rate. Furthermore, the private sector will reduce real capital per worker throughout its disequilibrium transition periods. The effects on private capital accumulation and the rate of inflation will be reversed if saving initially increases by more than the increase in hoarding following an increase in public capital per head. Real cash balances and real disposable income per person, however, may increase or decrease in the new stationary state. This ambiguity disappears when the government collects no direct fees from the private users of public capital, so that private firms employ such services up to their own saturation level. In this circumstance, only the first set of results is valid because initially this policy affects neither real output nor the real rate of return on the private capital but the tax burden is increased.

All of the preceding results were obtained by employing two extreme pricing rules with respect to the use of public capital services; namely, the government charges the direct users for the associated external diseconomies or the direct fee of using such services is zero. We have shown that in both of these cases the use rate of public capital services chosen by private firms is uniquely determined by the existing stock of public capital. In general, however, the use rate of public capital will also depend on the quantity of privately-supplied factors if the government sets a unit user's fee at some arbitrary level. Nonetheless, it can be shown that our main conclusions remain valid in the more general case.

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