Loss Factors Estimation of Statistical Energy Analysis Using Power Injection Method

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The statistical energy analysis (SEA) is an effective method to predict noise and vibration in the high-frequency band. To predict vibration and noise accurately by using SEA, it is important to estimate parameters called loss factors and modal density. The power injection method (PIM) is an effective experimental method to estimate SEA parameters accurately. In this experimental method it is necessary to take a lot of vibration measurement points because the number of measurement point influences the estimated result using PIM. However, there is no concrete guideline concerning the number of measurement points. In addition, it is necessary to measure the vibration in all subsystems to estimate parameters when the structure is changed. In this paper, we verified the influence of the number of measurement points on the analytical result. Furthermore, as an example of structural modification, we estimated the loss factors of a structure when attached with damping material using the experimental result of a single subsystem. As a result, we can present the concrete guideline concerning decision of the number of measurement point and it is possible to omit the process of estimating the loss factors after a structural modification using the proposed technique in the study.

Key words: Statistical Energy Analysis, Power Injection Method, Loss Factors, Measurement Points

1. Introduction

In recent years, reciprocating internal combustion engine is widely used as the power source for industrial machinery. This engine produces high level of vibration and noise especially during combustion process, which has been a particular issue. Thus there is a need for a prediction method to efficiently reduce vibration and noise emission. Prediction method using Finite Element Method (FEM) and Boundary Element Method (BEM) are effective for low frequency vibration analysis. However, in order to analyze structure-borne noise and vibration it is necessary to consider the overall audible frequency range including the high frequency range noise and vibration which could be predicted more effectively using Statistical Energy Analysis (SEA).

SEA method which was first introduced in the 1960's by Lyon et.al, as a response prediction method for acoustic and vibration system of aerospace sector, is an effective method to predict high frequency range of noise and vibration¹⁾. In order to predict the vibration, it is necessary to estimate the SEA parameters called damping loss factors (DLF) and coupling loss factors (CLF).The loss factors could be experimentally estimated by conducting excitation tests on a single subsystem²⁻⁵⁾. However, for a complex structure, it is difficult to estimate the loss factors by conducting single subsystem excitation tests. In order to tackle this

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problem, Power Injection Method (PIM) was proposed where it is possible to evaluate the loss factors for complex structure even when all the subsystems are connected together ⁶⁾. When conducting the analysis, a system is divided into simplified subsystems in order to estimate the parameters accurately. In addition, because the numbers of measurement points affects the accuracy of parameters estimation and vibration prediction, multiple points of vibration measurements are carried out during experiments. However, there is no guidance in determining specific measurement points. In addition, when there was a structural modification on a subsystem, parameters had to be estimated again on the entire system, thus increasing the number of experiments that needs to be conducted.

This paper aims to verify the influence of vibration measurement points on the vibration response prediction results. Furthermore, we proposed and verified the validity of damping loss factors estimation technique for a structure when a single subsystem was attached with damping materials as an example of structural modification.

2. Theory of SEA

2.1 Overview

The Statistical Energy Analysis is a prediction method of sound and vibration for complex structures or system that is divided into several subsystems and characterized by quantities of stored vibration energy and modes within narrow frequency bands. The balance equation between input power, power dissipation and transmission power for each two subsystems can be described as power flow balance equation. Furthermore, in order to solve the equation, it is required to analyze the vibration state of the subsystems and calculate the parameters such as loss factors.

2.2 Power flow balance equation for multiple subsystems

The power flow balance equation for N number of subsystems for a particular system can be described in matrix equation which can be written by the following equation⁷⁾;

where ω is the angular frequency, η_i is the damping loss factors, η_{ij} is the coupling loss factors, N_i is the mode number, E_i is the subsystem energy and P_i is the input power. The energy of each subsystem can be obtained by this equation if the loss factors matrix, which is the second term on the left-hand side, is given. Therefore obtaining an accurate loss factors is significant.

The structure subsystem energy is calculated by the following equation by the spatial average of vibration velocity v and mass m;

$$E = M \left\langle v^2 \right\rangle \tag{2-2}$$

Here, $\langle v^2 \rangle$ is the spatial root mean square of vibration velocity. With Eq. (2-2), the subsystem's vibration can be calculated if energy is obtained from a power balance Eq. (2-1).

3. Calculation of Loss Factors

3.1 Power Injection Method (PIM)

PIM simultaneously estimates damping and coupling loss factors⁸⁾. In this method, vibration power is injected into each subsystem to measure the vibration energy in each subsystem. Each loss factors is estimated by using these experimental data. The coupling loss

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Fig. 1. Picture of box-shaped structure

factors are estimated by the following equation;

$$\eta_{ij} \cong \frac{1}{\omega} \frac{\langle E_{ji} \rangle}{\langle E_{ii} \rangle} \frac{P_j}{\langle E_{jj} \rangle}$$
(3-1)

where $\langle \rangle$ shows the root mean square value. This equation is constituted by the energies of the focused and conterminous subsystems. The damping loss factors are estimated using the following equation;

$$\eta_{i} = \frac{P_{i}/\omega - (\sum E_{ii}\eta_{ii}) + (\sum E_{ji}\eta_{ji})}{E_{ii}}$$
(3-2)

3.2 Decay Ratio Method

The decay ratio method calculates the time history of the damped vibration⁹⁾. The subsystem is excited and after the excitation stops, the damping vibration is measured. The logarithmic decrement is calculated from the measured signal. The damping loss factors can be calculated from the following equation using the reverberation time, which is defined as the time at which the energy decays by 60 dB;

$$\eta = \frac{2.2}{T_{60} \cdot f}$$
(3-3)

Where η is the damping loss factors, *f* is the frequency, and T_{60} is the reverberation time.

4. Test Object

Fig.1 shows a test model that is constructed of a base, a roof, and four frames with thickness of 2.3mm, and three panels with thickness of 1.6mm. The external size of the test model is $700 \times 500 \times 390$ mm, and its



Fig. 2. SEA model

structural subsystems are fixed with M8 bolts.

The object is divided into some subsystems in SEA based on the following assumptions;

1) The bended part of the subsystems and the shin panels deal with equivalent thickness.

2) Screw holes are neglected.

Fig. 2 shows the SEA model. Subsystem 1 is the base, subsystem 2 is the roof, subsystems 3-6 are the frames, and subsystems 7-9 are the panels.

5. The Effect of Measurement Points to Loss Factors

5.1 Experimental method

We conducted an excitation experiments using power injection method in order to calculate the damping loss factors and coupling loss factors. Each subsystem is excited using Wilcoxon Research F3 shaker, and the vibration response is measured using Polytec Laser Doppler vibrometer. One random point on each subsystem was excited using sweep-sine wave signal with the range of 100 to 6000 Hz. The measurement points of vibration response for base and roof is 130 points each, frame 19 points each, panel 7 and panel 9 is 80 points each, and panel 8 is 150 points. 5.2 The variance of mean average values on the change of average measurement points

When identifying the loss factors, several points from the measured points from each subsystem were taken and the averaged values are considered as space average.

The mean average values of the vibration velocity



Fig. 3. Damping loss factors identification result

were calculated from the randomly extracted measurement points and the standard deviation of the mean average for the different number of measurements were calculated as variance. As a result, it is confirmed that the variance of the mean average becomes smaller by taking more measurement points. Therefore, it is thought that the average number of measurement points does affect the results of loss factors identification.

5.3 The change of loss factors for the change of average measurement points

5 random points from the measured values were taken and the damping loss factors are identified. The calculations for the identification of damping loss factors were done 100 times for different random points. The calculated standard deviation of the identified damping loss factors for the base and roof are shown in Fig. 3. Based on the figure we could see that there is variance on the damping loss factors. Therefore, it is clear that the average measurement points can affect the damping loss factors identification result.

5.4 The relationship between average measurement points and vibration prediction results

A model applied with the identified loss factors was created using the averaged 5 points of the average vibration velocity. Using the model that was created, the response of each subsystem was predicted during base excitation. Fig. 4 shows the combination of the standard deviation of prediction results for panel 8, actual measurements and calculated results. As shown



Fig. 4. Estimation result of vibration response



Fig. 5. Comparison between measured and identified results

in Fig. 4, variance occurs in the analysis result even though the vibration responses are well predicted.

Fig. 5 shows the variance of analysis results for panel 8 with different average measurement points. As shown in Fig. 5, the variance of analysis results was reduced with less average measurement points. It could be considered that the occurrence of variance in the mean average values where the average measurement points are few will result in the occurrence of variance in the vibration prediction result.

5.5 Predicting the effect of variance in average measurement value on the vibration prediction results

The effect of variance in measurement values for 3 subsystems model analysis on vibration prediction result was predicted. The power flow equation for 3 subsystems can be represented by the following equation,

where η_i is the damping loss factors, η_{ij} is the coupling loss factors, E_i is the subsystem energy and P_i is the input power. From Eq. (5-1), the energy of each subsystem can be calculated from the following formula,

$$\begin{cases} E_{I} \\ E_{2} \\ E_{3} \end{cases} = \frac{1}{\omega} \begin{bmatrix} \eta_{1} + \eta_{12} & -\eta_{21} & 0 \\ -\eta_{12} & \eta_{2} + \eta_{21} + \eta_{23} & -\eta_{32} \\ 0 & -\eta_{23} & \eta_{3} + \eta_{32} \end{bmatrix}^{-1} \begin{cases} P_{I} \\ P_{2} \\ P_{3} \end{cases}$$
(5-2)

Furthermore, by using the equations for calculating damping loss factors and coupling loss factors from the power injection method, each loss factors can be redefined as energy. Therefore, the calculation of damping loss factors for 3 subsystems can be given by the following equation,

$$\eta_{i} = \frac{P_{i}/\omega - (E_{ii}\eta_{ij} + E_{ii}\eta_{ik}) + (E_{ji}\eta_{ji} + E_{ki}\eta_{ki})}{E_{ii}}$$
(5-3)

In addition, because the coupling loss factors can only be implemented on two adjacent subsystems, by substituting Eq. (5-3) and Eq. (3-1) into Eq. (5-2), it can be written by the following expression,

$$\{E_a\} = \frac{1}{\omega} [E_m]^{-1} \{P'\}$$
(5-4)

where, $\{E_a\}$ is the energy vector of the analysis results, $[E_m]$ is the energy matrices of measurement results and $\{P'\}$ is the input power vector. In this case, because there is no significant variance in input power, its effect can be ignored and the variance of the measurement values was analyzed. Considering ΔE_{ij} as the variance of each measurement values, the variance of analyzed values $\{\Delta E_a\}$ can be written as,

$$\{\Delta E_a\} = \frac{1}{\omega}$$

$$\times \left[\frac{\partial}{\partial E_{II}} [E_m]^{-1} \Delta E_{II} + \dots + \frac{\partial}{\partial E_{ij}} [E_m]^{-1} \Delta E_{ij}\right]$$

$$\times \{P'\}$$
(5-5)

Table 1. Partial differential coefficient

	$\angle E_{I}$	ΔE_7	$\angle E_2$
$\frac{\partial}{\partial E_{II}} [E_m]^{-1}$	1.0087	0.00890	0.00326
$\frac{\partial}{\partial E_{77}} [E_m]^{-1}$	0.0384	0.0183	-0.0774
$\frac{\partial}{\partial E_{22}} [E_m]^{-1}$	0.00604	0.00501	0.0042
$\frac{\partial}{\partial E_{17}} [E_m]^{-1}$	-0.0750	-0.0442	-0.0161
$\frac{\partial}{\partial E_{7I}} [E_m]^{-1}$	-0.0849	0.89133	0.32415
$\frac{\partial}{\partial E_{72}} \left[E_m \right]^{-1}$	-0.0115	-0.0077	-0.0234
$\frac{\partial}{\partial E_{27}} \left[E_m \right]^{-1}$	-0.0183	0.00560	0.2114

Here, the effect on vibration prediction result of variance for each measurement values could be analyzed.

Equation (5-5) was applied to 3 subsystems of base, panel 7 and roof. For 2500Hz, the differential coefficient calculation result of Eq. (5-5) is shown in Table 1. Measurement energy used for the differential coefficient calculation is the actual measured energy.

According to Table 1, we could identify the measurement values that have the highest contribution of variance in each energy values. The approximation equation using the measurement values with high contribution of variance can be shown as,

$$\Delta E_{I} = \frac{\partial}{\omega \partial E_{II}} \left[E_{m} \right]^{-1} \left\{ P' \right\} \Delta E_{II}$$
(5-6)

$$\Delta E_2 = \frac{\partial}{\omega \partial E_{71}} \left[E_m \right]^{-1} \left\{ P' \right\} \Delta E_{71} + \frac{\partial}{\omega \partial E_{27}} \left[E_m \right]^{-1} \left\{ P' \right\} \Delta E_{27}$$
(5-7)

The calculated variance of energy values using Eq. (5-6) and Eq. (5-7), and the variance of analyzed energy values during base excitation using power injection method are shown in Fig. 6.







Based on Fig. 6, it is understood that the variance of analyzed values could be estimated accurately from the actual measurement values with high contribution of variance. Therefore, by implementing this approach, the variance of analyzed values could be estimated before conducting power injection method, and the number of measurement points could be determined.

Loss factors change estimation method for a single subsystem

It is understood that by attaching damping material on a structure, the damping loss factors of the subsystem would change. However, in order to utilize power injection method to calculate the loss factors, it is necessary to measure the entire structure all over



(a) Shape (b) Cross-Sectional Surface Fig. 7. Damping material



again. Therefore, we conduct excitation experiments only on subsystem that was attached with damping materials and estimated loss factors based on the different amount of attached damping materials. In addition, we also estimated the damping loss factors for mounted structure. The damping loss factors of a single subsystem were estimated using decay ratio method.

6.1 Experimental method

In order to calculate the damping loss factors using decay ratio method, we conducted hammering test and measured the free vibration. The measurement and excitation points are each 5 points and the panel was hanged to simulate free support condition during experiment.

Rubber material was used for the damping material. The cross-sections and outline of the rubber material is shown in Fig. 7.The damping material is attached to panel 8 with 2 different ways which are diagonally across the panel and squared around the panel. The lengths for diagonal attachment are 1.0m, 0.5m, 0.25m, 0.12m, and the lengths for squared attachment are 2.0m, 1.64m, 1.36m, 1.0m, 0.76m, and

0.40m. The attachment diagram is shown in Fig. 8.6.2 Loss factors estimation of a single subsystem

In this section will clarify the relationship between the length of attached damping material and the amount of change on damping loss factors. In addition, we derived an estimation equation of damping loss factors for the changes in the length of damping material.

It was reported that the damping loss factors for a flat plate is a constant regardless of the dimension of the plate ⁷). In a similar way, it could be considered that the damping loss factors of the damping material are not related to the length. In fact however, the damping loss factors do changes depending on the length of the damping material. This is because when energy was flowing through the panel attached with damping material also changes. Therefore, we estimated the damping loss factors of the panel attached the damping material and the panel.

During the energy flow through the panel attached with damping material, the panel and damping material each are assumed as a single subsystem, and the damping energy loss can be represented by the following equation,

$$\eta_{all}E_{all} = \eta_1 E_1 + \eta_2 E_2 \tag{6-1}$$

where, η_{all} is the damping loss factors of panel attached with damping material, E_{all} is the energy of panel attached with damping material, η_1 is the damping loss factors of panel, E_l is the panel's energy, η_2 is the damping loss factors of damping material, E_2 is the energy of damping material. From Eq. (6-1) η_{all} can be written as the following equation,

$$\eta_{all} = \frac{\eta_I E_I + \eta_2 E_2}{E_{all}} \tag{6-2}$$

where, $E_{all}=E_1+E_2$. η_{all} can be calculated from the ratio between E_1 and E_2 .

Next, we could derive the relational equation between E_1 and E_2 from the power flow balanced equation of 2 subsystems when power is injected onto the panel. The power flow balanced equation is shown by the following equation,

$$\begin{cases} P_1 \\ P_2 \end{cases} = \omega \begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2 + \eta_{21} \end{bmatrix} \begin{cases} E_1 \\ E_2 \end{cases}$$
(6-3)

Form Eq. (6-3), energy can be expressed as follows,

$$\begin{cases} E_1 \\ E_2 \end{cases} = \frac{1}{\omega} \begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2 + \eta_{21} \end{bmatrix}^{-1} \begin{cases} P_1 \\ P_2 \end{cases}$$
(6-4)

Here, because the input power to the damping material $P_2=0$, Eq. (6-4) can be rewritten as follows,

$$\begin{cases} E_{I} \\ E_{2} \end{cases} = \frac{1}{\omega} \frac{P_{I}}{\eta_{I}\eta_{2} + \eta_{2}\eta_{I2} + \eta_{I}\eta_{2I}} \begin{cases} \eta_{2} + \eta_{2I} \\ \eta_{I2} \end{cases}$$
(6-5)

From Eq. (6-5) we could obtain,

$$E_1 = \frac{\eta_2 + \eta_{21}}{\eta_{12}} E_2 \tag{6-6}$$

By substituting Eq. (6-6) into Eq. (6-2), η_{all} could be calculated from the following equation,

$$\eta_{all} = \frac{\eta_1 \eta_2 + \eta_1 \eta_{21} + \eta_2 \eta_{12}}{\eta_2 + \eta_{21} + \eta_{12}}$$
(6-7)

From Eq. (6-7), it is possible estimate the damping loss factors of panel attached with damping material if the damping loss factors of panel and damping material, and the coupling loss factors between panel and damping material are known. η_1 was calculated using decay ratio method, whereas a constant value of η_2 = 0.1 which is a common value of rubber material for all frequency range was used because it is difficult to obtain η_2 experimentally ¹⁰. In addition, coupling loss factors can be theoretically calculated from the following equation¹¹,

$$\eta_{ij} = \frac{C_{gi}L_c\tau_{ij}}{\pi\omega S_i} \tag{6-8}$$

where, C_{gi} is the bending-wave group velocity of subsystem *i*, which can be represented by phase velocity C_{bi} , where $C_{gi}=2C_{bi}$. Moreover, L_c is the bond length, τ_{ij} is the energy transmittance from subsystem



Fig. 9. Optimized result of coupling loss factor

i to subsystem *j*, and S_i is the area of subsystem *i*. Even though Eq. (6-8) can be used to identify the coupling loss factors, it is theoretically difficult because of the complex cross-section rubber material. Therefore, from the experimental results of diagonal attachment for 1.0m and squared attachment for 2.0m, optimized calculation was conducted. The optimization used the following objective function *J*,

$$J = \left(\eta_{meas.} - \frac{\eta_1 \eta_2 + \eta_1 \eta_{21} + \eta_2 \eta_{12}}{\eta_2 + \eta_{21} + \eta_{12}}\right)^2$$
(6-9)

Where $\eta_{meas.}$ is the identified damping loss factors of the panel attached with damping material from the experiment. In order to minimize Eq. (6-9), η_{12} and η_{21} were optimized using quasi-Newton method. The optimization was conducted using ESTECO optimization tool, modeFRONTIER. Fig. 9 shows each optimized coupling loss factors. According to Eq. (6-8), η_{12} are proportional to the attachment length and thus it is possible to estimate η_{all} during the change in attachment length.

Fig. 10 and Fig. 11 show the estimated results using Eq. (6-7) and measured values for diagonal attachment and squared attachment respectively. From Fig. 10 and Fig. 11, by using the proposed method the damping loss factors during the change of attachment length could be estimated accurately.

6.3 Application on the system

The identified results of internal loss factors for a

single panel and the results using power injection method are shown in Fig. 12.

From the figure it is understood that the identification result was different for the same when it is in a structure form. Therefore, the identified value of η_1 of Eq. (6-7) from the power injection method was used to estimate the damping loss factors of structure form after the attachment of damping material. In addition, the optimized values of η_{12} and η_{21} from the previous section were used. The estimated result of



Fig. 10. Estimation result of damping loss factor for diagonal attachment



(a) 1.64 m







Fig. 12. Damping loss factor comparison result of decay ratio method and power injection method





damping loss factors of the structure with diagonal attachment using Eq. (6-7) and the identified values using power injection method are shown in Fig. 13.Based from the figure, the damping loss factors were able to be estimated accurately. In conclusion, by using the proposed technique, the amount of change in damping loss factors of a structure form when attached with damping material was able to be estimated.

7. Conclusions

In this paper, we estimated the loss factors using the power injection method. The following conclusions are drawn from this investigation;

- The variance on the average values using the power injection method shows the influence of the number of measurement points on the loss factors estimation results and the analysis results.
- 2) By using the variance of the average values from the measurement results of the elements which has the highest contribution, the actual variance of analysis results for the different average numbers were able to be estimated.
- By estimating the variance of analysis results prior to performing experiments using power injection method, a technique to determine the number of measurement points was proposed.

Furthermore, based on the experiment results of a single element with the attachments of damping materials, a technique to identify the damping loss factors of combined elements with the attachments of damping materials was proposed and validated.

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