Do Faster Growing Economies Run Current Account Deficits? A Theoretical Reappraisal of the Role of Utility Functions

Shingo Iokibe

Abstract

It is well recognized in the literature that economies that have higher output growth prospects run current account deficits and that economies that have slower output growth prospects run current account surpluses. This paper examines the robustness of this hypothesis in a two-country framework from a purely theoretical point of view and finds that the crucial presumption concerns variability in the elasticity of marginal utility with respect to consumption. It also finds that the level of economic development or relative per-capita output to the rest of the world is crucial.

Key words: Current account; Relative GDP growth rates; Elasticity of marginal utility; Two-period models

I Introduction

It is well recognized in the literature that economies that have higher output growth prospects run current account deficits and that economies that have slower output growth prospects run current account surpluses (the conventional hypothesis hereafter).

The logic behind this hypothesis comes from neoclassical growth theory. According to the neoclassical growth theory presented by Solow (1956), the main driver of per-capita output growth is productivity growth in the economy as a whole. An economy that has a relatively high productivity growth rate attracts foreign capital because of its relatively high marginal rate of return on capital; thus, it imports physical capital in order to invest into future productions and runs a current account deficit. The international difference in the rate of return on capital, or that in the per-capita real output growth rate, determines the direction of net international capital flows in this framework.

However, capitalists that have physical capital and claims for returns on it, are consumers, too. Therefore, they make decisions not only about in which country to invest their capital but also about how much output they can lend to or borrow from abroad. This implies that they must consider which country is the more profitable for investing capital and assess how their consumption paths over time will be affected by their decisions. This inference suggests that the sign of the current account depends not only on international differences in the rate of return on capital but also on the consumption function and the expected path of consumption.

Surprisingly, few previous studies have attached importance to the latter point. Instead, most have simply assumed that if a country has a relatively higher rate of return on capital and thus a relatively higher output growth prospect, it consumes a part of future outputs in advance by borrowing current outputs from foreign countries. For example, Gruber and Kamin (2007) wrote that "(a)n increase in the growth rate of productivity relative to other countries should be associated with a more negative current account balance, as an increase in *the return on capital increases investment and the potential for higher future income decreases saving*" (p.505, italic added by the author).

This argument is based on the small country assumption. However, in a two-country model, the hypothesis indicated by the italics above may not always hold because higher future income in a low-income or less developed country can be consistent with a limited decline in its marginal utility. Then, a net import of physical capital and a net export of current consumption can occur simultaneously, and thus the sign of the current account becomes ambiguous.[1]

The present paper bridges this gap in the literature by exploring these problems. Specifically, it examines what kinds of presumptions are necessary for the derivation of the conventional hypothesis by focusing on consumption determinations in a two-country model without capital formations. It finds that the crucial presumption concerns the concavity of the period utility function. If the period utility function has a constant elasticity of marginal utility or of intertemporal substitution, the relative expected growth rate between countries is the single determinant of the sign of the current account between them. In other words, the conventional hypothesis holds. However, if the utility function has an increasing or decreasing elasticity of marginal utility, the conventional hypothesis does not always hold.

Further, if the utility function has an increasing elasticity of marginal utility, faster growing less developed countries can run a current account surplus or slower growing more developed countries can run a current account deficit. This situation occurs because the marginal utility in a low-income country drops by a lesser degree from period 1 to period 2 than that in a high-income country.

The remainder of this paper is organized as follows. Section II develops a general two-

1　We cannot find any extended reference to this issue in the voluminous text by Obstfeld and Rogoff (1996), which explains the relation between the relative growth rate and the current account mainly by using small country models.

country two-period model and describes its basic settings. Section III analyzes the determinants of the sign of the current account in period 1 in our proposed model. We then clarify under which kinds of conditions the conventional hypothesis holds; namely, we explore when faster growing economies run current account deficits. In section IV, we select empirically plausible cases from the results attained in section III. Section V investigates in depth the cases in which the conventional hypothesis does not hold. Section VI concludes.

Ⅱ The Model

In this section, we develop a one-good, two-period, two-country, perfect foresight model in a general form. We ignore the formation of the physical capital of firms and governmental activities throughout the analysis in this paper in order to focus on how international differences in the output growth rate influence the current account.

Let us assume a world economy that consists of country A and country B. In each country, representative households are born at the dawn of period 1, with populations N_A and N_B, respectively. These representative households live for two periods only. On birth, they realize that they are determined to receive real output $y_{k,t}$ at the beginning of each period, but no other economic shocks occur throughout either period.

Under the assumption of perfect foresight, representative households maximize their lifetime utilities at the beginning of period 1 by choosing their volumes of consumption in both periods. Let us set the lifetime utility function in an additive form.

$$(1)\quad u(c_{k,1})+\beta_k u(c_{k,2}).$$

$c_{k,t}$ is real consumption for the representative household in country k in period $t (k=A, B; t=1, 2)$. $u(\cdot)$ is the period utility function, which is assumed to satisfy $u'(\cdot)>0$ and $u''(\cdot)<0$ and be common to households in both countries. β_k is the subjective discount factor for country k's representative agent and we assume $0<\beta_k<1$.

The international capital market is perfectly unified. Representative households in either country can lend or borrow real products without paying any transaction costs. Further, although intra-national capital markets are perfectly unified in both countries, the homogeneous agent assumption prevents them from lending to and borrowing from each other in the domestic capital market.[2] The intertemporal budget constraint for the representative agent can be de-

2 Because households in one country have identical initial assets, endowments in each period, subjective ↗

fined as follows:

(2)　$b_{k,t}=(1+r_{t-1})b_{k,t-1}+y_{k,t}-c_{k,t}$.

$b_{k,t}$ denotes the net foreign assets of country k's representative agent at the end of period t. She or he receives (or repays) the principal and the interest of the net asset (or debt) at the beginning of period $t+1$.

Any debt is assumed to be repaid by the end of period 2.

(3)　$b_{A,2}=b_{B,2}=0$.

For expositional purposes, without loss of generality, we express output in country B using the parameter α throughout the rest of this paper. Let us denote y as the period 1 endowment of country A's representative household or the per-capita gross domestic products (GDP) of country A, while αy ($\alpha>0$) corresponds to the said value for country B. From equations (2) and (3), by expressing g_k as the real per-capita GDP growth rate of country k from period 1 to period 2, the intertemporal budget constraints for households in both countries are described as equations (4) and (5), respectively.

(4)　$$c_{A,1}+\frac{1}{1+r_1}c_{A,2}=(1+r_0)b_{A,0}+y+\frac{1+g_A}{1+r_1}y$$

and

(5)　$$c_{B,1}+\frac{1}{1+r_1}c_{B,2}=-(1+r_0)\frac{N_A}{N_B}b_{A,0}+\alpha y+\frac{1+g_B}{1+r_1}\alpha y.$$

We used the fact that net foreign assets sum to zero across the world in order to derive equation (5); i.e., $N_Ab_{A,t}+N_Bb_{B,t}=0$.

From equations (1), (4), and (5), we attain familiar first-order conditions for the utility maximization problem of representative agents.

(6)　$$\frac{u'(c_{A,1})}{\beta_A u'(c_{A,2})}=\frac{u'(c_{B,1})}{\beta_B u'(c_{B,2})}=1+r_1.$$

Under perfect international capital mobility, the marginal rates of the substitution of in-

↘ discount rates, and utility functions, they have no incentive to make an intertemporal trade with each other.

tertemporal consumption are equalized between countries and equal the world real interest rate, $1+r_1$.

However, we need the market equilibrium conditions in order to close the model. These outputs are assumed to be nondurable, and they have to be consumed in the period in which they are produced. Because the assumption of positive marginal utility excludes a leftover, the sum of consumptions in both countries equals the volume of world output or endowment in each period[3]:

$$(7) \quad N_A c_{A,1} + N_B c_{B,1} = (N_A + N_B \alpha) y$$

and

$$(8) \quad N_A c_{A,2} + N_B c_{B,2} = [N_A(1+g_A) + N_B(1+g_B)\alpha] y.$$

Ⅲ Growth Prospects and the Current Account

We assume throughout this paper that initial net foreign assets are zero ($b_{A,0} = b_{B,0} = 0$). This assumption makes the analytical process identical to analyzing the effects of the liberalization of the capital account. There is consensus in the literature that the emergence of current account imbalances after capital account liberalization depends on whether a difference exists between the autarky interest rates in country A and country B, namely the interest rates prevailing before liberalization in the domestic markets of each country.

In a closed economy, representative agents consume all their own endowments (income) in each period or the condition $c_{k,t} = y_{k,t}$ always holds. In this case, the marginal rate of the substitution of intertemporal consumption becomes as follows:

$$(9) \quad 1 + r_1^{A,\,aut} \equiv \frac{u'(y)}{\beta_A u'((1+g_A)y)}$$

and

3 However, we do not use these market equilibrium conditions (7) and (8), because the main purpose of our paper is not to investigate the size of the current account in a model with a specific utility function, but rather to examine the determinants of the sign of the current account in a general model without specific utility functions.

$$(10)\quad 1+r_1^{B,\,aut}\equiv\frac{u'(\alpha y)}{\beta_B u'((1+g_B)\alpha y)}.$$

Here, we denote the autarky interest rate as $r_1^{k,\,aut}$ similar to in equation (6). Note that the gross ratio of $1+r_1^{k,\,aut}$ represents the marginal rate of substitution, or the relative (utility-based) price of period 1 consumption to period 2 consumption. However, this ratio does not mean that representative agents in one country actually lend to and borrow from each other at this interest rate. A higher value for $r_1^{k,\,aut}$ implies that the representative household regards period 1 consumption as scarcer in the autarky economy.

Therefore, when country A runs a current account surplus in period 1 after its capital account liberalization, the following inequalities must hold:

$$(11)\quad c_{A,1}=y-\frac{CA_{A,1}}{N_A}<y,\qquad c_{A,2}=(1+r_1)\frac{CA_{A,1}}{N_A}+(1+g_A)y>(1+g_A)y$$

and

$$(12)\quad c_{B,1}=\alpha y+\frac{CA_{A,1}}{N_B}>\alpha y,\qquad c_{B,2}=-(1+r_1)\frac{CA_{A,1}}{N_B}+(1+g_B)\alpha y<(1+g_B)\alpha y.$$

Here, $CA_{k,\,t}$ represents country k's current account in period t.[4] By using equations (6), (9), and (10), inequalities (11) and (12) can be rearranged into the following inequalities:

$$(13)\quad 1+r_1^{A,\,aut}<\frac{u'(c_{A,1})}{\beta_A u'(c_{A,2})}=\frac{u'(c_{B,1})}{\beta_B u'(c_{B,2})}<1+r_1^{B,\,aut}.$$

Inequalities (13) show that when the autarky interest rate in country A is less than that in country B, country A runs a current account surplus in period 1, while country B runs a deficit. If the autarky interest rate in country A is lower than that in country B, country B's households can sacrifice more period 2 goods for period 1 goods compared with the volume of period 2 goods that country A's households need to acquire in compensation for period 1 consumption. Therefore, once international capital transactions are freed, both countries' households can raise their lifetime utilities by exchanging with each other period 1 consumption for period 2 consumption at an interest rate between the two autarky rates. In this case, country A runs a current account surplus or lends period 1 goods, while country B runs a current account deficit or borrows period 1 goods. Note that if the autarky interest rates in both countries are accidentally equalized, the current accounts must be zero because no households

4　A plus sign for $CA_{k,\,t}$ represents a current account surplus.

can find an interest rate at which they can trade period 1 consumption for period 2 consumption in order to improve their utilities in a sense of Pareto efficiency.

The above discussion can be summarized as the relations between the difference in the autarky interest rates and the sign of the period 1 current account. That is,

$$(14)\quad \begin{cases} r_1^{A,\,aut} < r_1^{B,\,aut} & \Leftrightarrow \quad CA_{A,1} > 0 \\ r_1^{A,\,aut} = r_1^{B,\,aut} & \Leftrightarrow \quad CA_{A,1} = 0. \\ r_1^{A,\,aut} > r_1^{B,\,aut} & \Leftrightarrow \quad CA_{A,1} < 0 \end{cases}$$

Because this paper aims to examine the necessary conditions under which a current account surplus (or deficit) occurs in a framework of a two-period model, we next investigate the determinants of the difference between $r_1^{A,\,aut}$ and $r_1^{B,\,aut}$. Equations (9) and (10) show that the relative magnitudes of $r_1^{A,\,aut}$ and $r_1^{B,\,aut}$ depend on g_A, g_B, α, β, and β_B.

Ⅲ–1 In the Case of $g_A = g_B$ and $\alpha = 1$

When the per-capita GDP growth rate and its level are identical for both countries, the country that has the smaller subjective discount factor (β) has a lower autarky interest rate and thus runs a current account surplus in period 1.[5]

Ⅲ–2 In the Case of $\beta_A = \beta_B$ and $g_A = g_B$ ($\alpha \neq 1$)

Next, we consider cases in which the subjective discount rate and per-capita GDP growth rate are identical for both two countries, but their initial levels of per-capita GDP are different. If we denote $\beta_A = \beta_B = \beta$ and $g_A = g_B = g$, the autarky interest rates reduce to the following simplified forms:

$$(15)\quad 1 + r_1^{A,\,aut} = \frac{u'(y)}{\beta u'((1+g)y)}$$

and

$$(16)\quad 1 + r_1^{B,\,aut} = \frac{u'(\alpha y)}{\beta u'((1+g)\alpha y)}.$$

Let us set $f(c) \equiv \dfrac{u'(c)}{\beta u'(ac)}$, where parameter a is a constant and satisfies $a > 0$. The deriva-

5 See Obstfeld and Rogoff (1995, 1996).

tive of f is

$$(17)\quad f'(c) \equiv \frac{u''(c)}{\beta u'(ac)}\left[1 - \frac{\varepsilon(ac)}{\varepsilon(c)}\right].$$

Here, $\varepsilon(c)$ is the elasticity of marginal utility with respect to consumption.

$$(18)\quad \varepsilon(c) \equiv -\frac{u''(c)c}{u'(c)} > 0.$$

From equations (17) and (18), when $a > 1$ holds, there exist the following relationships between $\varepsilon'(c)$ and $f'(c)$:[6]

$$(19)\quad \begin{cases} \varepsilon'(c) < 0 & \Leftrightarrow \quad f'(c) < 0 \\ \varepsilon'(c) = 0 & \Leftrightarrow \quad f'(c) = 0 \\ \varepsilon'(c) > 0 & \Leftrightarrow \quad f'(c) > 0 \end{cases}$$

.

By comparing relations (19) with equations (15) and (16), we can conclude that in the case of $\alpha > 1$ the relative size of the autarky interest rates are determined by the sign of $\varepsilon'(c)$. That is, the following relationships hold:

$$(20)\quad \begin{cases} \varepsilon'(c) < 0 & \Leftrightarrow \quad r_1^{A,\,aut} > r_1^{B,\,aut} \\ \varepsilon'(c) = 0 & \Leftrightarrow \quad r_1^{A,\,aut} = r_1^{B,\,aut}. \\ \varepsilon'(c) > 0 & \Leftrightarrow \quad r_1^{A,\,aut} < r_1^{B,\,aut} \end{cases}$$

Even in the case of $\alpha < 1$, the point of discussion is entirely the same. If $\alpha < 1$, the inequalities on the right-hand side of (20) become reversed, because now country A does not produce a smaller but rather a larger per-capita GDP than country B.[7]

In summary, in the case of a symmetric per-capita GDP growth rate and subjective discount rate, the determinants of the current account are the degree of economic development in terms of per-capita GDP and the sign of $\varepsilon'(c)$:

---

6 Recall that we have assumed $u'(c) > 0$ and $u''(c) < 0$.

7 The results in the text depend on the assumption of $g > 0$. When $g = 0$, the autarky interest rates in both countries are identical to β^{-1}, and thus, the current account becomes zero. When $g < 0$, the relationships in (20) are reversed: i.e., if $\varepsilon'(c) < 0$ holds, $r_1^{A,\,aut} < r_1^{B,\,aut}$ holds, and vice versa. However, because it is implausible that the world economy as a whole does not grow, we dismiss these cases from our discussion.

- If $\varepsilon'(c)=0$, the current account becomes zero irrespective of the level of economic development, or living standards.
- If $\varepsilon'(c)>0$, the less developed country runs a current account surplus.
- If $\varepsilon'(c)<0$, the less developed country runs a current account deficit.

The reason for this finding is as follows. If the period utility function satisfies $\varepsilon'(c)=0$, the marginal utility of period 2 consumption declines relative to that of period 1 consumption at the constant rate ε, irrespective of the volume of per-capita consumption in period 1. However, if $\varepsilon'(c)>0$ holds, the marginal utility of period 2 consumption in a less developed country, namely an economy that has a smaller per-capita income, declines relative to the marginal utility of period 1 consumption by a lesser extent than that in a more developed country does. Therefore, the marginal rate of the substitution of period 1 consumption with period 2 consumption becomes lower in the less developed country. On the contrary, if $\varepsilon'(c)<0$ holds, in a less developed country, the marginal utility of period 2 consumption declines relative to that of period 1 consumption by a larger extent than that in the more developed country does, and thus, the marginal rate of the substitution becomes higher.

This argument can be clarified algebraically. The inverses of equations (15) and (16) can be rearranged as below when the endowment growth rate is sufficiently small:

$$(21)\quad \frac{\beta u'((1+g)y)}{u'(y)} \cong \beta[1-\varepsilon(y)g]$$

and

$$(22)\quad \frac{\beta u'((1+g)\alpha y)}{u'(\alpha y)} \cong \beta[1-\varepsilon(\alpha y)g].$$

If $\alpha>1$ holds, the right-hand side of (21) equals the right-hand side of (22) only when $\varepsilon'(c)=0$ holds. However, the right-hand side of (21) becomes larger than that of (22) if $\varepsilon(c)>0$ holds and vice versa.

Intuitively, the condition of $\varepsilon'(c)>0$ means that households in a less developed country derive more utility from a one-percentage point increase in period 2 consumption than those in a higher developed country. Stated differently, the agents in a less developed economy evaluate future consumption more than the agents in a more developed economy do. This finding seems to be realistic.

Moreover, the utility gain derived from a marginal increase in consumption expenditure from an expenditure level of 120 US dollars (USD) per month is not much less than that derived from a marginal consumption increase from a level of 100 USD per month. However, marginal utility at an expenditure level of 4800 USD per month might be significantly less than that at 4000 USD per month even though the consumption growth rate (in terms of expenditure) is 20% in both examples.[8] If this argument were true, the assumption of the constant elasticity of intertemporal substitution, $\varepsilon'(c)=0$, is too simplistic to compare the dynamics of the national savings or current accounts between countries that show significant differences in per-capita GDP.

Ⅲ−3 In the Case of $\beta_A=\beta_B$ and $g_A\neq g_B$

Ⅲ−3−1 In the Case of $\alpha>1$

Next, we investigate the case of different per-capita GDP growth rates, $g_A\neq g_B$. First, let us assume $\alpha>1$. We discuss those cases when $\alpha\leq 1$ in sections III−3−2 and III−3−3.

(a) In the Case of $1+g_A\geq(1+g_B)\alpha$

First, the following inequality always holds if $1+g_A\geq(1+g_B)\alpha$ holds:

$$(23)\quad \frac{u'(y)}{\beta u'((1+g_A)y)}>\frac{u'(\alpha y)}{\beta u'((1+g_B)\alpha y)}.$$

This can easily be seen by recalling that the inequality $1+g_A>(1+g_B)\alpha$ is equivalent to

$$(24)\quad \frac{u'(y)}{u'(\alpha y)}>1\geq\frac{u'((1+g_A)y)}{u'((1+g_B)\alpha y)}.$$

Inequality (23) tells us that country A runs a current account deficit if $1+g_A\geq(1+g_B)\alpha$ holds. Note that this assumption implies $g_A>g_B$ under the presumption of $\alpha>1$.

In the case of $1+g_A<(1+g_B)\alpha$, we must examine the two cases of $g_A<g_B$ and $g_A>g_B$ separately.

(b) In the Case of $1+g_A<(1+g_B)\alpha$ and $g_A<g_B$

In this case, $r_1^{A,\,aut}$ and $r_1^{B,\,aut}$ become as follows:

$$(25)\quad 1+r_1^{A,\,aut}\equiv\frac{u'(y)}{\beta u'((1+g_A)y)}$$

8　We assume implicitly that the inflation rate is zero in both these examples.

and

$$(26)\quad 1+r_1^{B,\,aut}\equiv\frac{u'(\alpha y)}{\beta u'((1+g_B)\alpha y)}.$$

Using the conditions of (19), we can state that if $g_B\geq 0$ and $\varepsilon'(c)\geq 0$, the following inequalities hold:

$$(27)\quad \frac{u'(y)}{\beta u'((1+g_A)y)}<\frac{u'(y)}{\beta u'((1+g_B)y)}\leq\frac{u'(\alpha y)}{\beta u'((1+g_B)\alpha y)}.$$

By contrast, if parameter a in (17) is less than one, the relation between the signs of $\varepsilon'(c)$ and $f'(c)$ becomes in an inverse form of that in (19). In other words, in the case of $a<1$, the following relationships hold:

$$(28)\quad \begin{cases}\varepsilon'(c)<0 & \Leftrightarrow\quad f'(c)>0\\ \varepsilon'(c)=0 & \Leftrightarrow\quad f'(c)=0.\\ \varepsilon'(c)>0 & \Leftrightarrow\quad f'(c)<0\end{cases}$$

Using the relations presented in (28) allows us to verify that if $g_A\leq 0$ and $\varepsilon'(c)\leq 0$, the following inequalities hold:

$$(29)\quad \frac{u'(y)}{\beta u'((1+g_A)y)}\leq\frac{u'(\alpha y)}{\beta u'((1+g_A)\alpha y)}<\frac{u'(\alpha y)}{\beta u'((1+g_B)\alpha y)}.$$

In short, when $g_A<g_B$ holds, $r_1^{A,\,aut}$ is always smaller than $r_1^{B,\,aut}$, or country A runs a current account surplus, either if both $g_B\geq 0$ and $\varepsilon'(c)\geq 0$ hold or if $g_A\leq 0$ and $\varepsilon'(c)\leq 0$ hold.

(c) In the Case of $1+g_A<(1+g_B)\alpha$ and $g_A>g_B$

The relations in (19) suggest that when $g_A\geq 0$ and $\varepsilon'(c)\leq 0$ hold, the following inequalities hold:

$$(30)\quad \frac{u'(y)}{\beta u'((1+g_A)y)}\geq\frac{u'(\alpha y)}{\beta u'((1+g_A)\alpha y)}>\frac{u'(\alpha y)}{\beta u'((1+g_B)\alpha y)}.$$

In a similar manner, the relations in (28) suggest that the following inequalities hold when $g_B\leq 0$ and $\varepsilon'(c)\geq 0$ hold:

$$(31)\quad \frac{u'(y)}{\beta u'((1+g_A)y)} > \frac{u'(y)}{\beta u'((1+g_B)y)} \geq \frac{u'(\alpha y)}{\beta u'((1+g_B)\alpha y)}.$$

In short, country A runs a current account deficit because $r_1^{A,\,aut} > r_1^{B,\,aut}$ holds if one of the following conditions holds:

- $1+g_A \geq (1+g_B)\alpha$
- $1+g_A < (1+g_B)\alpha$, $g_A \geq 0$ and $\varepsilon'(c) \leq 0$
- $1+g_A < (1+g_B)\alpha$, $g_B \leq 0$ and $\varepsilon'(c) \geq 0$

Ⅲ−3−2 In the Case of $\alpha = 1$

When $\alpha = 1$ holds, the relative size of the autarky interest rates depends solely on the relative real per-capita GDP growth rate, $(1+g_A)$ / $(1+g_B)$, as is evident in equations (25) and (26). In other words, country A runs a current account surplus when its output growth rate is below that of country B and vice versa.

The theoretical results discussed in sections III−2, III−3−1, and III−3−2 are summarized in rows (1) to (5) of Table 1.

Table 1　Relations between the economic growth rate and the current account when $B_{A,0}=0$ and $\beta_A=\beta_B$

				(a) $g_A < g_B$	(b) $g_A = g_B$[1]	(c) $g_A > g_B$
(1)	$\alpha = 1$			$CA_{A,1} > 0$	$CA_{A,1} = 0$	$CA_{A,1} < 0$
(2)	$\alpha > 1$	$\frac{1+g_A}{1+g_B} \geq \alpha$		n.a.[2]	n.a.[2]	$CA_{A,1} < 0$
(3)		$\frac{1+g_A}{1+g_B} < \alpha$	$\varepsilon'(c) = 0$	$CA_{A,1} > 0$	$CA_{A,1} = 0$	$CA_{A,1} < 0$
(4)			$\varepsilon'(c) > 0$	$CA_{A,1} > 0$, if $g_B \geq 0$	$CA_{A,1} > 0$, if $g > 0$ $CA_{A,1} < 0$, if $g < 0$	$CA_{A,1} < 0$, if $g_B \leq 0$
(5)			$\varepsilon'(c) < 0$	$CA_{A,1} > 0$ if $g_A \leq 0$	$CA_{A,1} < 0$, if $g > 0$ $CA_{A,1} > 0$, if $g < 0$	$CA_{A,1} < 0$, if $g_A \geq 0$
(6)	$\alpha < 1$	$\frac{1+g_A}{1+g_B} \leq \alpha$		$CA_{A,1} > 0$	n.a.[3]	n.a.[3]
(7)		$\frac{1+g_A}{1+g_B} > \alpha$	$\varepsilon'(c) = 0$	$CA_{A,1} > 0$	$CA_{A,1} = 0$	$CA_{A,1} < 0$
(8)			$\varepsilon'(c)0$	$CA_{A,1} > 0$, if $g_A \leq 0$	$CA_{A,1} < 0$ if $g > 0$ $CA_{A,1} > 0$, if $g < 0$	$CA_{A,1} < 0$, if $g_A \geq 0$
(9)			$\varepsilon'(c) < 0$	$CA_{A,1} > 0$, if $g_B \geq 0$	$CA_{A,1} > 0$ if $g > 0$ $CA_{A,1} < 0$, if $g < 0$	$CA_{A,1} < 0$, if $g_B \leq 0$

1) When $g_A = g_B = 0$, $CA_{A,1} = 0$ always holds.
2) When $g_A \leq g_B$ and $\alpha > 1$ holds, $(1+g_A) \leq (1+g_B)\alpha$ always holds.
3) When $g_A \geq g_B$ and $\alpha < 1$ holds, $(1+g_A) \geq (1+g_B)\alpha$ always holds.

Ⅲ–3–3 In the Case of $\alpha<1$

In the case of $\alpha<1$, the relations between the sign of the current account, the relative per-capita output growth rate, and the concavity of the utility function are essentially the same as the case for $\alpha>1$. The results are shown in rows (6) to (9) of Table 1. By interchanging term A with term B and α with α^{-1} in rows (6) to (9) in Table 1, we attain the same results as those presented in rows (2) to (5).

Ⅲ–4 Summary of the Theoretical Results

As shown in Table 1, roughly speaking, the relative growth rates between countries determine the sign of the current account in line with the conventional hypothesis. Specifically, a faster (slower) growing country runs a current account deficit (surplus).

However, our theoretical result does not exclude exceptions, depending on the concavity of the period utility function, the extent of international differences in the per-capita real GDP growth rate, and the sign of the per-capita real GDP growth rate in individual countries.

As summarized in Table 2, the sign of the current account becomes indeterminate in several cases. Under the assumption of $\alpha>1$, if one of the conditions (a)–(d) in Table 2 holds, the sign of the current account is uncertain in a general model. For example, the current account can be either in surplus or in deficit if all the conditions of $(1+g_A)<(1+g_B)\alpha$, $\varepsilon'(c)<0$, and $0<g_A<g_B$ hold, as indicated in row (a). In a similar manner, under the assumption of $\alpha<1$, the sign of the current account balance is indeterminate if one of the conditions (e)–(h) applies. As in Table 1, the conditions (a)–(d) are essentially the same as the conditions (e)–(h), respectively.

Table 2　Indeterminate cases on the sign of the current account

(a)	$\alpha>1$	$\frac{1+g_A}{1+g_B}<\alpha$	$\varepsilon'(c)<0$	$0<g_A<g_B$
(b)				$0>g_A>g_B$
(c)			$\varepsilon'(c)>0$	$g_A<g_B<0$
(d)				$g_A>g_B>0$
(e)	$\alpha<1$	$\frac{1+g_A}{1+g_B}>\alpha$	$\varepsilon'(c)<0$	$g_A>g_B>0$
(f)				$g_A<g_B<0$
(g)			$\varepsilon'(c)>0$	$0>g_A>g_B$
(h)				$0<g_A<g_B$

Ⅳ Sorting by Data

The results presented in Tables 1 and 2 may appear to be too complicated to digest. By examining the real data, however, we can dismiss some of the cases in these tables from the perspective of plausibility.

Ⅳ−1 Size of α for Developed and Developing Countries

If we regard country B as the rest of the world (ROW) other than country A, and by using the nominal GDP values in USD from the World Bank's World Development Indicators (2009), α is calculated as 0.16 when country A is assumed to be the United States, while α equals 2.94 when country A is assumed to be China, for example. As shown in row (1) of Table 3, α is always less than one for all developed countries,[9] but larger, sometimes much greater, than one for developing countries.[10] In other words, the per-capita real GDP values of

Table 3 Summary statistics on the 10-year growth rates of the ROW, 1980–2009

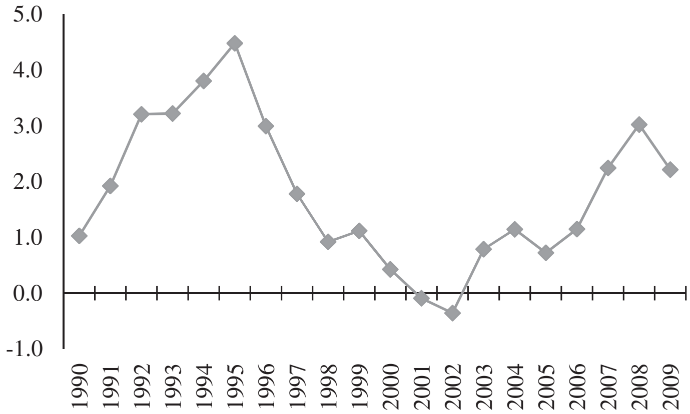
	Developed countries	Developing countries
(1) $\alpha \geq 1$	0 (0%)	1336 (94.5%)
(2) Average of α	0.32	9.8
(3) St. Dev. of α	0.17	10.4
(4) $\frac{1+g_A}{1+g_B} < \alpha$	1 (0.2%)	1344 (95.0%)
(5) $\frac{1+g_A}{1+g_B} \geq 1$	440 (68.0%)	613 (43.4%)
(6) Average of $1+g_A$	2.74	0.57
(7) Observations	647	1414

Source: Author's calculation using data from the World Development Indicators of the World Bank
Note: α and g_B are calculated by assuming that country B is the ROW other than country A. The numbers in parentheses are the percentage ratios to the total observations.

9 Developed countries include the following 33 countries: Australia, Austria, Belgium, Canada, Cyprus, Denmark, Finland, France, Germany, Greece, Hong Kong, Iceland, Ireland, Italy, Japan, Malta, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom, the United States, the Bahamas, Bahrain, Barbados, Israel, Kuwait, Oman, and Saudi Arabia.

10 Developing countries include the following 72 countries: Algeria, Argentina, Bangladesh, Belize, Benin, Bolivia, Botswana, Brazil, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Chile, China, Colombia, Republic of Congo, Cote d'Ivoire, Dominican Republic, Ecuador, Egypt, El Salvador, Fiji, Gabon, the Gambia, Ghana, Guatemala, Haiti, Honduras, India, Indonesia, Iran, Jamaica, Jordan, Kenya, Republic of Korea, Lesotho, Madagascar, Malawi, Malaysia, Mali, Mauritania, Mauritius, Mexico, Morocco, Nepal, Niger, Nigeria, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Rwanda, Senegal, the Seychelles, Sierra Leone, South Africa, Sri Lanka, Swaziland, Syrian Arab Republic, Tanzania, Thailand, Togo, Tunisia, Turkey, Uganda, Uruguay, Venezuela, Zambia, and Zimbabwe.

Figure 1 10-year per-capita real GDP growth rate since 1980



Source: Author's calculation using data from the World Development Indicators of the World Bank

Note: The unit is percentage. The presented numbers are world averages over 105 countries of the 10-year annualized growth rate of real per-capita GDP. Specifically, the 10-year real per-capita GDP growth rate at year t is the annualized growth rate from year t-10 to year t, calculated according to the method described in footnote 11.

developed countries exceed those of the ROW, while most of those of developing countries do not.

Ⅳ–2 Stable Relationship between α and the Relative Growth Rate

Based on data on the past 30 years, row (4) in Table 3 shows that $(1+g_A)/(1+g_B)>\alpha$ usually holds for developed countries, while $(1+g_A)/(1+g_B)<\alpha$ usually holds for less developed countries.[11] In other words, $(1+g_A)/(1+g_B)<\alpha$ usually holds in the case of $\alpha>1$, while $(1+g_A)/(1+g_B)>\alpha$ usually holds in the case of $\alpha<1$. Actually, 97.3% of observations with $\alpha>1$ satisfy $(1+g_A)/(1+g_B)<\alpha$, while 93.8% of observations with $\alpha<1$ meet $(1+g_A)/(1+g_B)>\alpha$.

Ⅳ–3 The ROW's Growth Rate is Positive

Figure 1 depicts the world average of the real per-capita GDP growth rates of the ROW for

11 To calculate $(1+g_A)/(1+g_B)$, we first calculated the annualized gross growth rate of the per-capita nominal GDP of country A and of the ROW over 10 years. Then, we divided the former by the latter. Specifically, we calculated the annualized growth rate of the per-capita nominal GDP from 1980 to 1990, $G_{1980-1990}$, for example, by using the following equation:

$$1+G_{1980-1990}=\left(\frac{Y_{1990}^{current\$}}{Y_{1980}^{current\$}}\right)^{\frac{1}{10}}.$$

Here, $Y_t^{current\$}$ denotes nominal GDP at year t evaluated in current USD amounts. These nominal growth rates allow us to calculate the relative real growth rate from the following formula:

$$\left(\frac{1+g_A}{1+g_B}\right)_{1980-1990}=\frac{1+G_{A,1980-1990}}{1+G_{B,1980-1990}}.$$

each country over the past 30 years. Although the long-run ROW growth rate swung to a large degree, it remained greater than zero in most years of this study period. On average, the ROW's real per-capita GDP growth rate for an individual country was[12] 1.8%. Although its 10-year average growth rates through 2001 and 2002 declined into a slightly minus range, we can interpret this as a result of the extraordinary global economic slowdown in these two years. In summary, we can say with certainty that the ROW's long-run real GDP growth rate has been positive over the past 30 years.

Ⅳ–4 Realistic Cases

These observations make it possible for us to ignore rows (2) and (6) in Table 1, because the conditions of $(1+g_A)/(1+g_B)>\alpha$ with $\alpha>1$ and of $(1+g_A)/(1+g_B)<\alpha$ with $\alpha<1$ rarely hold in the real world. We can also dismiss column (c) of rows (4) and (9) in Table 1 as well as rows (b), (c), (f), and (g) in Table 2, because $g_B<0$, namely a less-than-zero growth rate for the ROW, rarely seems to occur.

Next, we discuss the case of $(1+g_A)/(1+g_B)<\alpha$ with $\alpha\geq 1$ or equivalently the case of $(1+g_A)/(1+g_B)>\alpha$ with $\alpha\leq 1$; namely, the case when less developed economies grow at a relatively slow rate. In such a case, if either $\alpha=1$ or $\varepsilon'(c)=0$ holds, the relative size between g_A and g_B is the very factor that determines the sign of the current account. If neither of these conditions holds, the relation between the international relative growth rate and the sign of the current account becomes complex. In the case of $\varepsilon'(c)>0$, for example, a less developed economy can run a current account surplus even though it grows faster compared with the ROW. In other words, under some conditions, the current account of an individual country can always be in surplus or in deficit regardless of its relative growth rate compared with the ROW.

Similarly, the main determinant of the current account is the relative size of the marginal substitution rate of intertemporal consumption in autarky economies, which depends largely on the relative rates of the per-capita GDP growth rates between countries. If country A and country B are initially homogeneous in terms of per-capita GDP ($\alpha=1$) or if the period utility functions have a constant elasticity of marginal utility ($\varepsilon'(c)=0$), the relative output growth rate is the only factor that determines the relative size of the marginal substitution rate or the current account balance. However, once the period utility functions have a variable elasticity of marginal utility in the case of $\alpha\neq 1$, the marginal substitution rate is affected not

12 This value is the cross-country average of the country's mean of the 10 year annualized real per-capita GDP growth rate between 1980 and 2009.

only by the sign of the relative growth rate but also by the signs of the individual real per-capita GDP growth rates of each country. Next, we investigate this case more thoroughly.

V Unconventional Cases

As discussed in section III–2, $\varepsilon'(c)>0$ seems to be more plausible than $\varepsilon(c)<0$ because marginal utility at a lower consumption level is supposed to decrease to a lower degree than that at a higher consumption level. By examining two types of utility functions that satisfy $\varepsilon'(c)>0$, this section explores the conditions under which the conventional hypothesis that faster growing economies run current account deficits does not hold.

V–1 CARA Utility

First, let us consider a period utility function that has constant absolute risk aversion (i.e., the CARA utility function). The CARA utility function can be defined as

$$(32)\quad u(c) = -A\sigma\exp(-c/\sigma).$$

Here, A and σ are both positive and constant. The elasticity of marginal utility for this function is as follows:

$$(33)\quad \varepsilon(c) = -\frac{u''(c)c}{u'(c)} = -\frac{-\sigma^{-1}A\exp(-c/\sigma)c}{A\exp(-c/\sigma)} = \frac{c}{\sigma}.$$

From equation (33) we can easily see $\varepsilon'(c)>0$.

The autarky interest rates under this type of utility function are thus reduced to very simple forms.

$$(34)\quad 1 + r_1^{A,aut} = \frac{1}{\beta}\exp\left(\frac{g_A y}{\sigma}\right)$$

and

$$(35)\quad 1 + r_1^{B,aut} = \frac{1}{\beta}\exp\left(\frac{g_B \alpha y}{\sigma}\right).$$

From equations (34) and (35), it can easily be seen that in the case of $\alpha>1$, country A runs a current account surplus even when $g_A>g_B$ applies, if

(36)　$g_A < g_B \alpha$

holds. Note that condition (36) includes $(1+g_A) < (1+g_B)\alpha$. In other words, in case (d) in Table 2, if the relative size and relative growth rate of the home country's per-capita GDP to the ROW's per-capita GDP satisfy condition (36), faster growing economies can run a current account surplus contrary to the conventional hypothesis.

By contrast, in the case of $\alpha < 1$, country A runs a current account deficit even when it grows slower than does the ROW, if

(37)　$g_A > g_B \alpha$

holds. This is an example of case (h) in Table 2.

In short, either in the case of $g_B < g_A < g_B \alpha$ with $\alpha > 1$ or in the case of $g_B \alpha < g_A < g_B$ with $\alpha < 1$, the current account in period 1 has the opposite sign to the conventional hypothesis. This result is interesting in that no condition is subject to the parameters of the CARA utility function, A and σ.

Ⅴ–2 Quadratic Utility

The next example of a utility function that has $\varepsilon'(c) > 0$ is a quadratic utility function such as

(38)　$u(c) = -c^2 + 2mc \ (m > 0,\ 0 < c \leq m).$

We can easily verify $\varepsilon'(c) > 0$ based on $\varepsilon(c) = c\,/\,(m-c)$. The autarky interest rates are thus specified in the following form:

(39)　$$1 + r_1^{A,\,aut} = \frac{m - y}{\beta[m - (1+g_A)y]}$$

and

(40)　$$1 + r_1^{B,\,aut} = \frac{m - \alpha y}{\beta[m - (1+g_B)\alpha y]}.$$

Comparing equation (39) with equation (40) determines that in the case of $\alpha > 1$, country A runs a current account surplus even when $g_A > g_B$ if condition (41) holds:

$$(41)\quad \frac{1+g_A}{1+g_B} < k = \frac{(m-\alpha y)+(m-y)g_B\alpha}{(m-\alpha y)(1+g_B)}.$$

As far as country A's period 1 per-capita GDP y is sufficiently small such that the following condition

$$(42)\quad y < \frac{m}{(1+g_B)\alpha}$$

holds, there exists k satisfying $1 < k < \alpha$.

In a similar manner, in the case of $\alpha < 1$, if country A's period 1 per-capita output falls in the range of (42), there exists $\tilde{k}$, which satisfies $\alpha < \tilde{k} < 1$ and

$$(43)\quad \tilde{k} = \frac{(m-\alpha y)+(m-y)g_B\alpha}{(m-\alpha y)(1+g_B)}.$$

If the relative growth rate is higher than $\tilde{k}$, or $(1+g_A) > (1+g_B)\tilde{k}$ holds, the current account of country A becomes a deficit even though it grows at a slower rate compared with the ROW.

In short, even in the case of a quadratic utility function, there exists some combination of g_A, g_B, α, and y with which a faster (slower) growing economy runs a current account surplus (deficit).

Ⅴ-3 Observation Breakdown

Table 4 describes how many observations were detected, in which the expectation of country A's per-capita real GDP growth rate falls in the range (36) or (37).

Here, we assume that representative households expect the economy to grow at the same rate as it did during the preceding decade. For example, at the beginning of 1991, the representative agent in country A regards both country A's and the ROW's per-capita economic

Table 4 Probability for the unconventional case in the case of the CARA utility

		(a) $g_B < g_A$	(b) $g_B < g_A < g_B\alpha$	(c) b/a
1)	Developing Country ($\alpha > 1$) ex African Countries	415	154	37.1%
2)	Developing Country ($\alpha > 1$) African Countries	198	141	71.2%
		(d) $g_A < g_B$	(e) $g_B\alpha < g_A < g_B$	(f) d/e
3)	Developed Country ($\alpha < 1$)	207	63	30.4%

Source: Author's calculation using data from the World Development Indicators of the World Bank growth rates in the subsequent decade to be identical to those between 1980 and 1990.

Under this assumption, we observed 415 year-country pairs for developing countries except African countries with which the corresponding country's growth prospect was higher than that for the ROW, namely $g_A > g_B$ holds. Among these observations, over one-third (154 observations) satisfied the condition (36), $g_B < g_A < g_B \alpha$. For African countries, a much larger proportion (70%) of observations with $g_A > g_B$ satisfied the condition (36). We also observed 207 year-country pairs for developed countries with which the corresponding country's growth prospect was lower than that for the ROW, namely $g_A < g_B$ holds. Among these observations, nearly one-third (63 observations) satisfied the condition (37), $g_B \alpha < g_A < g_B$.

These data suggest that under the assumption of CARA utility, the scenario in which a faster growing economy runs a current account surplus or a slower growing economy runs a current account deficit is plausible.

Ⅵ Concluding Remarks

When adopting two-period models, the hypothesis that a country growing relatively faster than the ROW runs a current account surplus is robust only when the period utility function has a *constant* elasticity of intertemporal substitution. However, once we assume that period utility functions have a *variable* elasticity of marginal utility, the hypothesis does not always hold. If we assume a utility function of which the elasticity of marginal utility increases with respect to consumption, less developed countries that grow faster compared with the ROW, can then run current account surpluses under selected conditions.

From a theoretical perspective, the present study shows that the sign of the current account in consumption-saving models depends solely on the relative magnitude of the intertemporal marginal substitution rate of consumption of the home country to the foreign country. Moreover, the marginal substitution rate is determined solely by the consumption growth rate and is unaffected by the initial level of consumption if the elasticity of marginal utility is constant. However, the marginal substitution rate is affected by the initial level of consumption as well as by the consumption growth rate if the elasticity of marginal utility is either increasing or decreasing with respect to consumption. These findings demonstrate why the conventional hypothesis cannot always hold when the utility function has a variable elasticity of marginal utility.

Because this study focused on examining what kind of hypothesis we could extract from the simplest models, we must point out some of the limitations of our theoretical results. Firstly, we ignored how initial net foreign asset holdings affect the current account. In the real world, many countries hold considerable net foreign assets or liabilities in terms of their share of GDP, which may influence the current account of the corresponding country in different ways. For example, Iokibe (2013) showed that positive initial net foreign assets might shrink current account surpluses.

Secondly, we focused on a two-period, one-good consumption model. Models that have more time periods, including indefinite time period models, may produce a different hypothesis because our theoretical results are largely affected by the two-period assumption that all debt will be repaid in period 2. Introducing multiple goods, such as home and foreign goods, makes it possible to investigate the effects of exchange rate movements. Moreover, our consumption-saving model cannot explain the dynamics of investment, the other major factor that determines the current account.

Thirdly, our model excludes uncertainty. Uncertainty about future output or future income might be a crucial factor that influences the saving behavior of households. Lastly, because our analysis targeted the determinants of the sign of the current account, we cannot state with certainty how the *size* of the current account is determined. Nevertheless, from the results summarized in Table 1, we can still conjecture that as the relative real per-capita growth rate of country A to country B increases, the more the current account deficit of country A grows. The above-mentioned issues should be explored and overcome in future research.

Acknowledgements: This paper was supported by (MEXT/JSPS) KAKENHI Grant Number 24730282 as well as by an overseas research grant from Doshisha University. This paper was accomplished during the author's stay at the Institute of East Asian Studies, University of California at Berkeley.

References

Gruber, Joseph W., and Steven B. Kamin (2007) "Explaining the Global Pattern of Current Account Imbalances," *Journal of International Money and Finance,* Vol.26, pp.500–522.

Iokibe, Shingo (2013), "Reexamining the Relationship between Net Foreign Assets and the Current Account," *Doshisha Shogaku,* Vol.64, No.6, pp.475–492.

Obstfeld, Maurice, and Kenneth Rogoff (1996), *Foundations of International Macroeconomics,* MIT Press.

Obstfeld, Maurice, and Kenneth Rogoff (1995), "The Intertemporal Approach to the Current Account," in Gene M. Grossman and Kenneth Rogoff eds, *The Handbook of International Economics,* Vol.3, North Holland, pp.1731–1799.

Solow, Robert (1956), "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics,* Vol.70, pp.65–94.

World Bank (2009), *World Development Indicators.*