

# An Analysis of Optimal Allocation and Accumulation of Cultural Capital

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## Abstract

One mean of bridging the gap between economy and culture is provided by the concept of cultural capital through which the economic analysis of cultural goods, services, behavior and other phenomenon can proceed. Cultural capital is originated from the capital given for the purpose of cultural use. This capital may be physical capital, human capital, natural capital or the mix of them. Such capital makes up an asset from where the flow of material and non-material cultural benefit arises. The question now arises: what is the socially optimal quantity of capital that should be made available for cultural use? In this paper, a model is developed in which cultural capital is represented, and the rules for the optimal allocation and accumulation of cultural capital are derived.

## 1. Introduction

One mean of bridging the gap between economy and culture is provided by the concept of cultural capital through which the economic analysis of cultural goods, services, behavior and other phenomenon can proceed. Cultural capital in an economic sense enables both tangible and intangible manifestations of culture to be articulated as long lasting stores of value and providers of benefits for individuals and groups (Throsby, 2001, p.44).

Throsby (1999) defined cultural capital as an asset that embodies or gives rise to cultural value in addition to whatever economic value it might possess. Cultural capital exists in two forms. First, cultural capital may be tangible such as buildings, locations, sites, precincts and artworks. Tangible cultural capital is created by human activities, lasts for a period of time, can decay if not maintained and increase through investment. Second, cultural capital may be intangible such as ideas, beliefs and values.

Only few attempts have so far been made at the implications of cultural capital for economic analysis. Ulibarri (2000) developed a theory of rational philanthropy in forming cultural capital and aesthetic preferences. Robson (2003) examined the term cultural capital in a wider context alongside its relationship with the other capitals and suggested that more attention to be given to the relationship that cultural capital has with economic and social capitals. Travaglini (2005) empirically tested for the existence of links between modern and ancient cultural capital stocks for a sample of over one hundred countries.

In most circumstances, for social, institutional and technological reasons, cultural capital is regarded as a common resource for collective consumption that cannot be privately appropriated (e.g., a historic building, a shrine and beliefs). Rüdiger and Sao (2000) identified that cultural capital exists to benefit all members of society. The stock of cultural capital is enlarged by the flow of new capital created by individuals who

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are both consumers and creators of culture and whose utility is positively affected by the cultural goods they created. This non-rivalry and non-excludability in use of cultural capital are the characteristics of public goods<sup>1</sup>.

Uzawa regarded cultural capital as an important component of Social Common Capital (SCC)<sup>2</sup>. SCC constitutes a vital element of any society in which we live. SCC is social equipments for all of us. It is generally classified into three categories: natural capital, social infrastructure, and institutional capital. Cultural capital, together with medical institutions, education institutions and others, is classified under institutional capital, which provides members of society with those services that are crucial in maintaining human and cultural life. SCC is held by the society as common property resources, to be managed by social institutions of various kinds that are entrusted on a fiduciary basis to maintain SCC in good order and to distribute the services derived from it equitably. SCC is in principle not appropriated to individual members of the society, without, however, precluding the private ownership arrangements (Uzawa, 2005, p. 5).

According to Ostrom and Ahn (2003, p.11), 'cultural' concept such as trust, trustworthiness and norms cannot be reduced from the ideas fundamental to the Social Capital approach. Social Capital, which was originally introduced by Coleman (1988) and Putnam (1993), refers to the aggregate actual and potential resources embedded in the social structures, including personal relations and formal network structures, which can be used to facilitate collective actions. These networks can generate trust, establish expectations, and create and enforce the norms

that facilitate further transactions. Most forms of Social Capital are in a fundamentally different position with other forms of capital. Coleman (1988) noted that Social Capital is a public good that benefit the society as a whole. On the contrary, Fukuyama (1999, p.256) argued that Social Capital will in fact be produced by private markets because it is in the long-term interests of selfish individuals to produce it. This study is consistent with Coleman's view. Intangible cultural capital (e.g., social norms, traditions, beliefs and values) does not benefit primarily a person but benefit all those who are part of the society.

One of the distinctive aspects of natural capital<sup>3</sup> was seen by ecological economists to be its sustainability property (Costanza and Daly, 1992). The concept of sustainable development<sup>4</sup> as defined in ecological terms can be extended to the sphere of culture by recognizing parallels between the concepts of natural and cultural capital.

Both natural and cultural capitals have been inherited from the distant or recent past, the former provided as a gift of nature, the latter deriving from human creativity. Both impose a duty of care on the present generation. Furthermore, just as natural ecosystems support the real economy, so do cultural systems. For example it can be argued that when cultural 'ecosystems' function well, human productivity can be increased and economic growth can be enhanced. Finally, the notion of diversity, which is of such overwhelming importance in the natural world, has an equally vital role to play in cultural systems; it is clear that cultural diversity makes an important contribution to artistic and cultural dynamism which, in turn, has flow-on effects in the economy (Throsby, 2001, pp. 51-52).

<sup>1</sup> The standard definition of public goods derives from a classic paper, only three pages long, by Samuelson (1954). A good is non-rival when the consumption of this good by one household does not reduce the quantity available for consumption by any other. A good is non-excludable when no household can be excluded from consuming it, once it is supplied.

<sup>2</sup> Uzawa (2005, p. 4) noted that "Cultural capital may also be regarded as an important component of SCC, as extensively examined in particular by Throsby (2001). Cultural capital comprises those capital assets in society that yield goods and services of cultural value, including artworks, historic buildings, and so on, together with intangible assets such as language, traditions and others".

<sup>3</sup> Uzawa (2005, p. 3) noted that "Natural capital consists of the natural environment and natural resources such as forests, rivers, lakes, wetlands, coastal seas, oceans, water, soil, and the earth atmosphere".

<sup>4</sup> Sustainable development is defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (World Commission on Environment and Development, 1987, p.43).

Subsequently, there has been some further interest in the notion of cultural sustainability, one strand of which was concerned with direct interactions between culture and the environment (Nassauer, 1997; Garcia Mira et al., 2003), another employing the recently developed concept of cultural capital (Throsby, 1997; Shockley, 2004) with a related strand applying sustainability concepts to urban cultural heritage (Strange, 1999; Rana, 2000).

The present paper identifies that cultural capital exists as a common resource of cultural goods and services, and provides the benefit both now and in the future. Cultural capital is originated from the capital given for the purpose of cultural use. This capital may be physical capital, human capital, natural capital or the mix of them. Such capital makes up an asset from where the flow of material and non-material cultural benefit arises. The question now arises: what is the socially optimal quantity of capital that should be made available for cultural use? In this paper, a model is developed in which cultural capital is represented, and the rules for the optimal allocation and accumulation of cultural capital are derived.

The paper is organized as follows. In the following section, we present Pareto efficient allocation of cultural capital from the view point of the social planner. We next investigate under what condition cultural capital is supplied in decentralized market. Given that cultural capital with less quantity is provided in a competitive economy, a tax-subsidy mechanism is introduced to achieve Pareto efficient equilibrium. In the next section, dynamic program developed by Uzawa (1974) is introduced to present the conditions under which capital accumulation over time is dynamically optimal.<sup>5</sup> Sustainable process of accumulation for cultural capital is explored from the viewpoint of the intergenerational distribution of utility. Furthermore, we recognize the existence of externality, which represents the connection between culture and economic behavior lying outside the price system. We next attempt to

obtain interpretations about the relationship between culture development and economic growth through examining dynamically optimal conditions. Finally, some conclusions are drawn.

## 2. Optimal Allocation of Cultural Capital

### 2.1 First Best Solution

Collective consumption of cultural capital reflects its feature of being a non-exclusive public good. We assume that no congestion effect exists for simplicity, which means that one does not reduce any other's utility through his/her own cultural activity. Hence our theoretical analysis may be based on theory of pure public goods, which implies non-excludability and non-rivalry of cultural capital (Samuelson, 1954).

We assume that there are two kinds of goods i.e., cultural goods and private goods, and two kinds of agents i.e., a finite number of households and perfectly competitive firms in the economy. Households are denoted by  $h = 1, \dots, H$ , who derive welfare from the consumption of both cultural and private goods. Each household's preference can be represented by a utility function,

$$u^h(c^h, g^h)$$

where  $c^h$  and  $g^h$  are respectively private and cultural goods consumed by individual household. The function is positive, continuously twice differentiable and strictly quasi-concave together with

$$u^h_{c^h} > 0, u^h_{g^h} > 0.$$

Cultural goods  $g^h$  is the outcome from cultural behavior, given by function,

$$g^h(l^h, G)$$

where  $l^h$  is the leisure time devoted in his/her cultural behavior and  $G$  is cultural capital. The total supply of cultural capital  $G$  appears in all household utility functions, which indicates

<sup>5</sup> Uzawa (1974) addressed two technical issues which were applied in the present paper: the Penrose effect and the constant impute price.

the pure public goods future of cultural capital. The function is positive, continuously twice differentiable, strictly quasi-concave and homogeneous of order one together with

$$g^h l^h > 0, g^h G > 0.$$

We assume that privately owned capital has two alternative uses: cultural and productive use. Individual contribution of capital in culture and production is respectively denoted by  $K_\alpha^h$  and  $K_\beta^h$ .

Cultural capital  $G$  with both culture value and economic value<sup>6</sup> comes from the capital given over to cultural use, denoted by  $K_\alpha$  where

$$K_\alpha = \sum_{h=1}^H K_\alpha^h.$$

The generation of culture value is presented by a production function,

$$G = G(K_\alpha)$$

which is positive and continuously twice differentiable together with

$$G' > 0, G'' < 0.$$

Finally, given the constant returns to scale for technology, we assume that a number of perfectly competitive firms can be considered as a single aggregate firm with the production function,

$$F(N, K_\beta)$$

where

$$N = \sum_{h=1}^H n^h$$

and

$$K_\beta = \sum_{h=1}^H K_\beta^h$$

are the total labor and capital devoted in production respectively. The function is positive, continuously twice differentiable, strictly quasi-concave and homogeneous of order one together with

$$F_N > 0, F_{K_\beta} > 0.$$

Constrained by total factors available and production possibility, an allocation of a set of goods, time and capital  $(c^h, l^h, n^h, K_\alpha^h, K_\beta^h)$  among

all feasible patterns of allocation is the Pareto efficient when the social utility is maximized. The maximization problem can be shown as

$$\max_{c^h, l^h, n^h, K_\alpha^h, K_\beta^h} \sum_{h=1}^H u^h(c^h, g^h)$$

subject to

$$\sum_{h=1}^H c^h = F(N, K_\beta) \quad (1)$$

$$l^h + n^h = \bar{T} \quad (2)$$

$$K_\alpha^h + K_\beta^h = \bar{K}^h \quad (3)$$

where constraint (1) states that total private goods consumption is constrained by production possibility. Equations (2) and (3) indicate that factors devoted in both cultural and productive uses are equal to the total amount available.

The Lagrangian for this maximization problem may be written as

$$L = \sum_{h=1}^H u^h(c^h, g^h(l^h, G(K_\alpha))) + \lambda_1 \left( F(N, K_\beta) - \sum_{h=1}^H c^h \right) + \sum_{h=1}^H \lambda_2 (\bar{T} - l^h - n^h) + \sum_{h=1}^H \lambda_3 (\bar{K}^h - K_\alpha^h - K_\beta^h)$$

where the unknown variables are  $c^h, l^h, n^h, K_\alpha^h$  and  $K_\beta^h$ . Lagrangian multipliers  $\lambda_1, \lambda_2$  and  $\lambda_3$  are respectively associated with constraints (1), (2) and (3). Partially differentiate the Lagrangian with respect to unknown variables and put them to zero, we get the first order conditions of Pareto efficient equilibrium:

$$\frac{\partial u^h}{\partial c^h} = \lambda_1 \quad (4)$$

$$\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial l^h} = \lambda_2 \quad (5)$$

$$\frac{\partial F}{\partial N} = \frac{\lambda_2}{\lambda_1} \quad (6)$$

$$\sum_{h=1}^H \frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial G} \frac{\partial G}{\partial K_\alpha} \frac{\partial K_\alpha}{\partial K_\alpha^h} = \lambda_3, \quad \forall h = 1, \dots, H \quad (7)$$

<sup>6</sup> In the context of cultural capital, a cultural asset may have economic value, which derives simply from its physical existence (Throsby, 1999).

$$\frac{\partial F}{\partial K_\beta} = \frac{\lambda_3}{\lambda_1}. \quad (8)$$

With the arrangement of equations (4), (5) and (6), we get

$$\frac{\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial l^h}}{\frac{\partial u^h}{\partial c^h}} = \frac{\partial F}{\partial N} \quad (9)$$

which states that an allocation of time is Pareto efficient when individual marginal rate of substitution between leisure and private goods consumption equals to the marginal productivity of labor.

Furthermore, with the arrangement of equations (4), (7) and (8), we get

$$\frac{\sum_{h=1}^H \frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial G}}{\frac{\partial u^h}{\partial c^h}} = \frac{\frac{\partial F}{\partial K_\beta}}{\frac{\partial G}{\partial K_\alpha}} \quad (10)$$

where the sum of marginal rate of substitution between cultural capital and private goods consumption over all households equals to the marginal rate of transformation between capital devoted in production and culture. Equation (10) can be viewed as a version of Samuelson rule, which states the Pareto optimal allocation of capital between cultural and productive use from the social point of view.

## 2.2 Private Provision under Decentralized Market System

Now that rules for Pareto efficient allocation of time and capital have been derived, the next question is whether the competitive economy can lead to this efficient result. In this section, the necessary conditions, which characterize Pareto efficient allocation, will be contrasted to those in decentralized economy in which the cultural capital is funded entirely by the voluntary contributions of individual household.

In a perfect competitive economy, the

household maximizes his/her own utility

$$u^h(c^h, g^h)$$

subject to

$$c^h = wn^h + rK_\beta^h \quad (11a)$$

$$l^h + n^h = \bar{T} \quad (11b)$$

$$K_\alpha^h + K_\beta^h = \bar{K}^h \quad (11c)$$

where equation (11a) states that individual private goods consumption is constrained by his/her wealth that comes from the payment for labor (at real wage) and for his/her capital invested in production (at real interest rate).

The Lagrangian for this maximization problem may be written as

$$L = u^h(c^h, g^h(l^h, G(K_\alpha))) + \lambda_1(wn^h + rK_\beta^h - c^h) + \lambda_2(\bar{T} - l^h - n^h) + \lambda_3(\bar{K}^h - K_\alpha^h - K_\beta^h)$$

where

$$K_\alpha = K_\alpha^h + \sum_{\substack{j=1 \\ j \neq h}}^H K_\alpha^j.$$

Partially differentiate the Lagrangian with respect to unknown variables and put them to zero, we get

$$\frac{\partial u^h}{\partial c^h} = \lambda_1 \quad (12)$$

$$\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial l^h} = \lambda_2 \quad (13)$$

$$w = \frac{\lambda_2}{\lambda_1} \quad (14)$$

$$\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial G} \frac{\partial G}{\partial K_\alpha} = \lambda_3 \quad (15)$$

$$r = \frac{\lambda_3}{\lambda_1}. \quad (16)$$

In a competitive economy the representative aggregate firm maximizes its returns by demanding labor and productive capital to produce private goods, characterized by

$$\max_{n^h, K_\beta^h} F(N, K_\beta) - wN - rK_\beta$$

The first order condition for this maximization problem is

$$\frac{\partial F}{\partial N} = w \quad (17)$$

$$\frac{\partial F}{\partial K_\beta} = r. \quad (18)$$

The equilibrium under decentralized market is a set of goods, time, capital and price of productive factors  $(c^h, l^h, n^h, K_\alpha^h, K_\beta^h, w, r)$  where  $(w, r)$  is equilibrium price when markets for private goods, labor and productive capital are clear and a set of  $(c^h, l^h, n^h, K_\alpha^h, K_\beta^h)$  is the solution to household's utility maximization problem.

With the arrangement of equations (12), (13), (14) and (17), we get

$$\frac{\frac{\partial u^h}{\partial c^h} \frac{\partial g^h}{\partial g^h}}{\frac{\partial u^h}{\partial c^h}} = \frac{\partial F}{\partial N}$$

which is same with condition (9), i.e., time as privately owned scarce resource is efficiently allocated under decentralized market.

With the arrangement of equations (12), (15), (16) and (18), we get

$$\frac{\frac{\partial u^h}{\partial c^h} \frac{\partial g^h}{\partial g^h}}{\frac{\partial u^h}{\partial c^h}} = \frac{\frac{\partial F}{\partial K_\beta}}{\frac{\partial G}{\partial K_\alpha}}$$

where individual marginal rate of substitution between cultural capital and private goods consumption equals to the marginal rate of transformation between capital in different uses. Since, individual household derives positive utility from both cultural and consumptive activities, such that

$$\frac{\frac{\partial u^h}{\partial c^h} \frac{\partial g^h}{\partial g^h}}{\frac{\partial u^h}{\partial c^h}} > 0 \quad \forall h = 1, \dots, H.$$

Hence, the sum of individual marginal rate of substitution among all households must be larger than the marginal rate of transformation, i.e.,

$$\frac{\sum_{h=1}^H \frac{\partial u^h}{\partial c^h} \frac{\partial g^h}{\partial g^h}}{\frac{\partial u^h}{\partial c^h}} > \frac{\frac{\partial F}{\partial K_\beta}}{\frac{\partial G}{\partial K_\alpha}}.$$

It can be observed that this condition is clearly different from the rule of efficiency presented in the previous section.

The cause of this inefficient result in the decentralized economy system is that each unit of capital invested in production is paid at real interest rate but capital endowed in culture is paid at zero. Moreover, cultural benefit arisen from individual capital contribution cannot be exclusively appropriated by his/her own. Hence, the free rider problem arises when individual household realizes that he/she can share the cultural benefit from others' contribution without being charged. Individual households may have an incentive to enjoy the benefits from cultural capital provided by others while providing it insufficiency. Given that this is true for each household, cultural capital with less quantity is provided in a competitive economy.

### 2.3 Lindahl Equilibrium

Since private contribution to cultural capital in decentralized market is scarce because of its low return, a tax-subsidy mechanism may be introduced to achieve Pareto efficient equilibrium in the market. Suppose there is another agent in economy called Cultural Capital Administration (CCA) authorized by central government. Each unit of capital endowed in culture will be subsidized by CCA at its opportunity cost, that is, the real interest rate or the marginal productivity

of productive capital. On the other hand, this subsidy is financed by the collection of a set of personalized tax. With the intervention of CCA utility, the maximization problem for individual households may be written as

$$\max_{c^h, l^h, n^h, K_\alpha^h, K_\beta^h, \theta} u^h(c^h, g^h)$$

subject to

$$c^h + \mu^h \theta = wn^h + rK_\alpha^h + rK_\beta^h \quad (19a)$$

$$l^h + n^h = \bar{T} \quad (19b)$$

$$K_\alpha^h + K_\beta^h = \bar{K}^h \quad (19c)$$

$$\theta = rK_\alpha. \quad (20)$$

In constraint (19a) each unit of capital contributed in cultural use is subsidized at its opportunity cost (real interest rate) and  $\mu^h \theta$  is the real personalized tax levied on him/her. Each household bears a  $\mu^h$  share of total tax income  $\theta$  where

$$\sum_{h=1}^H \mu^h = 1.$$

Equation (20) states that CCA's budget is always balanced.

The Lagrangian for this maximization problem may be written as

$$L = u^h(c^h, g^h) + \lambda_1 (wn^h + r(K_\alpha^h + K_\beta^h) - c^h - \mu^h \theta) + \lambda_2 (\bar{T} - l^h - n^h) + \lambda_3 (\bar{K}^h - K_\alpha^h - K_\beta^h) + \lambda_4 (rK_\alpha - \theta)$$

where the unknown variables are  $c^h, l^h, n^h, K_\alpha^h, K_\beta^h$  and  $\theta$ . Partially differentiate the Lagrangian with respect to unknown variables and put them to zero, we get the first order condition:

$$\frac{\partial u^h}{\partial c^h} = \lambda_1 \quad (21)$$

$$\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial l^h} = \lambda_2 \quad (22)$$

$$w = \frac{\lambda_2}{\lambda_1} \quad (23)$$

$$\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial G} \frac{\partial G}{\partial K_\alpha} + \lambda_1 r + \lambda_4 r = \lambda_3 \quad (24)$$

$$r = \frac{\lambda_3}{\lambda_1} \quad (25)$$

$$\mu^h = -\frac{\lambda_4}{\lambda_1}. \quad (26)$$

The first order condition for this profit maximizing aggregate firm is same with that in previous section

$$\frac{\partial F}{\partial N} = w \quad (27)$$

$$\frac{\partial F}{\partial K_\beta} = r. \quad (28)$$

With the arrangement of equations (21), (22), (23) and (27), we get

$$\frac{\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial l^h}}{\frac{\partial u^h}{\partial c^h}} = \frac{\partial F}{\partial N}$$

which is same with equation (9). This condition illustrates that time as a privately owned resource is efficiently allocated through competitive market with or without the intervention of CCA.

With the arrangement of equations (21), (24), (25), (26) and (28), we get

$$\frac{\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial G}}{\frac{\partial u^h}{\partial c^h}} = \mu^h \frac{\frac{\partial F}{\partial K_\beta}}{\frac{\partial G}{\partial K_\alpha}}. \quad (29)$$

Remember that  $\mu^h$  is the personalized tax share of total that sums to one. Add up (29) over all households, we get the condition that defines the Pareto efficient capital allocation:

$$\sum_{h=1}^H \frac{\frac{\partial u^h}{\partial g^h} \frac{\partial g^h}{\partial G}}{\frac{\partial u^h}{\partial c^h}} = \frac{\frac{\partial F}{\partial K_\beta}}{\frac{\partial G}{\partial K_\alpha}}.$$

We have several comments on this result. First, CCA does not provide capital for culture but its intervention corrects households' behavior towards an appropriate provision of cultural capital under decentralized market system.

Second, by solving (29) for  $\mu^h$  and substituting  $\partial F/\partial K_\beta$  with  $r$ , together with (20) we get the determination of personalized tax:

$$\mu^h \theta = \frac{\frac{\partial u^h}{\partial c^h} \frac{\partial g^h}{\partial K_\alpha} \frac{\partial G}{\partial K_\alpha}}{\frac{\partial u^h}{\partial c^h}} K_\alpha.$$

At given optimum  $K_\alpha$  personalized tax  $\mu^h \theta$  is determined by nothing but his/her marginal rate of substitution between cultural capital and private goods consumption. Since marginal rate of substitution may be interpreted as valuation, all households will be taxed according to his/her valuation on cultural capital.

Finally, the equilibrium with a set of personalized tax or so called personalized price is often called as Lindahl equilibrium (Lindahl, 1958) that demonstrates how efficient allocation of cultural capital can be attained in decentralized economy. Unfortunately, household's truly expressed valuation on cultural capital becomes the personalized tax he/she has to pay. Hence, in a competitive market system there is no incentive for individual household to report his/her valuation on cultural capital at real level. On the other hand, valuations on cultural capital may vary from person to person. CCA may be lack of knowledge of households' preferences or willingness to pay for cultural capital. In sum, Pareto efficient allocation of capital will not occur unless all households are supposed to express their preference honestly or, if not, CCA can properly identify them.

### 3. Sustainability and Cultural Capital

#### 3.1 Optimal Capital Accumulation

Cultural capital's long-lasting characteristics

reflecting the continuing and evolutionary nature of culture can be thought of within a framework provided by the concept of sustainability where intergenerational issues are especially prominent. Our conceptual framework developed in previous chapter will be extended to deal with the problem of dynamic optimum. For simplicity, assumptions in this chapter differ in some elements. Each household's utility function is

$$u^h(c^h(t), G(t))$$

where  $c^h(t)$  and  $G(t)$  are respectively the individual private goods consumption and the stock of cultural capital at time  $t$ . Utility function is positive, continuously twice differentiable and strictly quasi-concave together with

$$u^h_{c^h(t)} > 0, \quad u^h_{G(t)} > 0$$

Net national product from the given productive capital stock at time  $t$  is expressed by the production function,

$$Y = f(K_\beta(t)).$$

We assume it is positive and continuously twice differentiable together with

$$f' > 0, \quad f'' < 0.$$

The existence of the sustainable time path of capital accumulation starting with an arbitrarily given stock of capital is ensured when the processes of accumulation of kinds of capital are subject to Penrose effect. This concept of Penrose effect was originally introduced by Penrose (1959) to describe the growth process of an individual firm. It was formalized by Uzawa (1968, 1969) in the context of macroeconomic analysis of dynamic equilibrium to elucidate the effect of investment activities on the processes of capital accumulation and later was extended by Uzawa (1974) to the dynamic optimality of SCC. A particularly important concept associated with the Penrose effect is the decreasing marginal efficiency of investment. In our study, increasing marginal cost of investment is described in the terms of decreasing marginal efficiency of investment.



Let  $I_\beta(t)$  be the amount of real investment devoted to the accumulation of productive capital. The amount of real investment  $I_\beta(t)$  may not necessarily result in the increase in the stock of capital by the same amount. Instead, there exists a certain relationship between real investment  $I_\beta(t)$  and the corresponding increase  $\dot{K}_\beta(t)$  in the stock of productive capital  $K_\beta(t)$ . This relationship may be denoted by the Penrose function

$$I_\beta(t) = \phi_\beta(\dot{K}_\beta(t), K_\beta(t)). \quad (30)$$

Function (30) may be interpreted as follows: in order to increase the stock of productive capital  $K_\beta(t)$  by the amount  $\dot{K}_\beta(t)$ , real investment  $I_\beta(t)$  has to be spent on the accumulative activities. In what follows, it will be assumed that function  $\phi_\beta(\cdot)$  exhibits a feature of constant return to scale with respect to  $\dot{K}_\alpha(t)$  and  $K_\alpha(t)$ , thus we may write (30) as

$$\frac{I_\beta(t)}{K_\beta(t)} = \varphi_\beta\left(\frac{\dot{K}_\beta(t)}{K_\beta(t)}\right).$$

Since it may be assumed that the marginal cost of investment is increased, the function  $\varphi_\beta(\cdot)$  satisfies the following conditions

$$\varphi_\beta'(\cdot) > 0, \quad \varphi_\beta''(\cdot) < 0.$$

For simplicity we denote the rate of capital accumulation by  $z_\beta(t)$ , then

$$\frac{\dot{K}_\beta(t)}{K_\beta(t)} = z_\beta(t).$$

Similar relationships may be postulated for the accumulation of cultural capital. The amount of real investment  $I_\alpha(t)$  required to increase the stock  $K_\alpha(t)$  by the amount  $\dot{K}_\alpha(t)$  may be determined by the following function:

$$\frac{I_\alpha(t)}{K_\alpha(t)} = \varphi_\alpha\left(\frac{\dot{K}_\alpha(t)}{K_\alpha(t)}\right)$$

which satisfies the condition:

$$\varphi_\alpha'(\cdot) > 0, \quad \varphi_\alpha''(\cdot) < 0.$$

Since most cultural capital is unique or irreplaceable, any destruction of it may be a case of irreversible loss. People have obligations to spend on conservation and maintenance of cultural capital at present state. For simplicity, it is assumed that destruction of cultural capital is avoidable under certain preservation plan, but it may depreciate at a constant rate  $\gamma$  to current capital stock, denoted by the function:

$$D_\alpha(t) = \gamma K_\alpha(t).$$

The amount of real investment  $I_\alpha(t)$  may not only result in the increase in current stock of capital but also result in the maintenance of capital at current state, denoted by

$$\frac{I_\alpha(t)}{K_\alpha(t)} = \varphi_\alpha\left(\frac{\dot{K}_\alpha(t) + D(t)}{K_\alpha(t)}\right) = \varphi_\alpha\left(\frac{\dot{K}_\alpha(t)}{K_\alpha(t)} + \gamma\right).$$

Also we may simply write

$$\frac{\dot{K}_\alpha(t)}{K_\alpha(t)} + \gamma = z_\alpha(t).$$

The concept of sustainability involves intergeneration equity and justice. We assume that the intergeneration preference can be expressed by the utility integral (Cass, 1965)

$$\int_0^{+\infty} U(t) e^{-\delta t} dt$$

where

$$U(t) = \sum_{h=1}^H u^h(c^h(t), G(t))$$

and  $\delta$  is the rate of discount assumed to be positive and constant.

The feasibility constraints are

$$\sum_{h=1}^H c^h(t) + I_\alpha(t) + I_\beta(t) = f(K_\beta(t))$$

$$\dot{K}_\alpha(t) = (z_\alpha(t) - \gamma)K_\alpha(t)$$

$$\dot{K}_\beta(t) = z_\beta(t)K_\beta(t)$$

<sup>7</sup> We suppose that there is planner who invests in both cultural and productive capital, because our dynamic analysis is from the social point of view.

$$\begin{aligned} K_\alpha(0) &= K_\alpha^0 \\ K_\beta(0) &= K_\beta^0. \end{aligned}$$

The dynamically optimal path of the capital accumulation may be obtained as the optimal solution to the intergeneration utility maximization problem above.

The problem of dynamic optimum may be solved in terms of the Lagrange method. The Lagrangian for this maximization problem can be written as

$$\begin{aligned} L &= \int_0^{+\infty} \sum_{h=1}^H u^h(c^h(t), G(t)) e^{-\delta t} dt \\ &+ \int_0^{+\infty} \lambda_1(t) e^{-\delta t} \left( f(K_\beta(t)) - \sum_{h=1}^H c^h(t) - \varphi_\alpha(z_\alpha(t)) K_\alpha(t) - \varphi_\beta(z_\beta(t)) K_\beta(t) \right) dt \\ &+ \int_0^{+\infty} \lambda_2(t) e^{-\delta t} \left( (z_\alpha(t) - \gamma) K_\alpha(t) - \dot{K}_\alpha(t) \right) dt \\ &+ \int_0^{+\infty} \lambda_3(t) e^{-\delta t} \left( z_\beta(t) K_\beta(t) - \dot{K}_\beta(t) \right) dt \end{aligned}$$

where  $\lambda_1(t)$ ,  $\lambda_2(t)$  and  $\lambda_3(t)$  are the Lagrange multipliers at time  $t$ . The optimal solution at time may be obtained by partially differentiating the Lagrangian with respect to the unknown control variables  $c^h(t)$ ,  $z_\alpha(t)$  and  $z_\beta(t)$  and putting them equal to zero, then we get

$$\frac{\partial u^h(t)}{\partial c^h(t)} = \lambda_1(t) \quad (31)$$

$$\frac{\partial \varphi_\alpha(t)}{\partial z_\alpha(t)} = \frac{\lambda_2(t)}{\lambda_1(t)} \quad (32)$$

$$\frac{\partial \varphi_\beta(t)}{\partial z_\beta(t)} = \frac{\lambda_3(t)}{\lambda_1(t)}. \quad (33)$$

In accordance with Lagrangian function the Hamiltonian function can be written as

$$\begin{aligned} H &= \sum_{h=1}^H u^h(c^h(t), G(t)) e^{-\delta t} \\ &+ \lambda_1(t) e^{-\delta t} \left( f(K_\beta(t)) - \sum_{h=1}^H c^h(t) - \varphi_\alpha(z_\alpha(t)) K_\alpha(t) - \varphi_\beta(z_\beta(t)) K_\beta(t) \right) \\ &+ \lambda_2(t) e^{-\delta t} \left( (z_\alpha(t) - \gamma) K_\alpha(t) - \dot{K}_\alpha(t) \right) \\ &+ \lambda_3(t) e^{-\delta t} \left( z_\beta(t) K_\beta(t) - \dot{K}_\beta(t) \right) \end{aligned}$$

Solving for state variables  $K_\alpha(t)$  and  $K_\beta(t)$ , we get the dynamical equations for the imputed prices of capital for cultural and productive use

$$-\sum_{h=1}^H \frac{\partial u^h(t)}{\partial G(t)} \frac{\partial G(t)}{\partial K_\alpha(t)} + \lambda_1(t) \varphi_\alpha(z_\alpha(t)) - \lambda_2(t) (z_\alpha(t) - \gamma) = \dot{\lambda}_2(t) - \delta \lambda_2(t) \quad (34)$$

$$-\lambda_1(t) \left( \frac{\partial f(t)}{\partial K_\beta(t)} - \varphi_\beta(z_\beta(t)) \right) - \lambda_3(t) z_\beta(t) = \dot{\lambda}_3(t) - \delta \lambda_3(t). \quad (35)$$

Together with the transversality conditions,

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_2(t) K_\alpha(t) = 0$$

and

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_3(t) K_\beta(t) = 0$$

we have derived the first order conditions of the maximization problem above.

Equation (32) and (33) suggest that dynamically optimum investments in cultural and productive capital occur when  $\varphi_\alpha'(z_\alpha(t))$  and  $\varphi_\beta'(z_\beta(t))$  equal to their imputed prices  $\lambda_2(t)/\lambda_1(t)$  and at time  $\lambda_3(t)/\lambda_1(t)$  respectively. The imputed price of private goods  $\lambda_1(t)$  is characterized in equation (31), and the dynamic differential equations for  $\lambda_2(t)$  and  $\lambda_3(t)$  are given by equation (34) and (35).

With the arrangement of (31), (32) and (34), we get

$$\frac{\dot{\lambda}_2(t)}{\lambda_2(t)} = \delta - (z_\alpha(t) - \gamma) - \frac{\sum_{h=1}^H \frac{\partial u^h(t)}{\partial G(t)} \frac{\partial G(t)}{\partial K_\alpha(t)} / \frac{\partial u^h(t)}{\partial c^h(t)} - \varphi_\alpha(z_\alpha(t))}{\frac{\partial \varphi_\alpha(t)}{\partial z_\alpha(t)}}$$

which can be simply written as

$$\frac{\dot{\lambda}_2(t)}{\lambda_2(t)} = \delta + \gamma - z_\alpha(t) - \frac{r_\alpha(t) - \varphi_\alpha(z_\alpha(t))}{\frac{\partial \varphi_\alpha(t)}{\partial z_\alpha(t)}} \quad (36)$$

where  $r_\alpha(t)$  defined in (36) is the sum of marginal substitution between cultural capital and private goods consumption over all households. With the arrangement of (31), (33) and (35), we get

$$\frac{\dot{\lambda}_3(t)}{\lambda_3(t)} = \delta - z_\beta(t) - \frac{\frac{\partial f(t)}{\partial K_\beta(t)} - \varphi_\beta(z_\beta(t))}{\frac{\partial \varphi_\beta(t)}{\partial z_\beta(t)}}$$

which can be simply written as

$$\frac{\dot{\lambda}_3(t)}{\lambda_3(t)} = \delta - z_\beta(t) - \frac{r_\beta(t) - \varphi_\beta(z_\beta(t))}{\frac{\partial \varphi_\beta(t)}{\partial z_\beta(t)}} \quad (37)$$

where  $r_\beta(t)$  defined in (37) is the marginal productivity of productive capital. Equation (36) and (37) are the conditions for the imputed prices of cultural capital and productive capital respectively. The dynamic optimal patterns of accumulation for both cultural and productive capital may be described by condition (36) and (37) describing the rules by which the imputed prices change over time.

The existence of the dynamically optimum time-path satisfying these optimal conditions is generally not guaranteed. Uzawa (1974, 1988) stated that it is extremely difficult to find the dynamic optima when state variables are more than one. In order to approximate the structure of the optimum path of SCC, Uzawa (1974) considered the case where the impute prices of capital are assumed to be constant over time. In our study, the rate of accumulation for both

cultural and productive capital may be obtained by assuming that equations (36) and (37) are equated to zero, that is, the imputed prices are assumed constant over time. Hence, the rate of capital accumulation  $z_\alpha(t)$  and  $z_\beta(t)$  may be obtained by the following conditions

$$\frac{\partial \varphi_\alpha(t)}{\partial z_\alpha(t)} = \frac{r_\alpha(t) - \varphi_\alpha(z_\alpha(t))}{\delta + \gamma - z_\alpha(t)}$$

and

$$\frac{\partial \varphi_\beta(t)}{\partial z_\beta(t)} = \frac{r_\beta(t) - \varphi_\beta(z_\beta(t))}{\delta - z_\beta(t)}$$

The determination of the rate of accumulation,  $z_\alpha(t)$  and  $z_\beta(t)$  may be illustrated by Figure 1 and Figure 2 respectively through which we may have a qualitative analysis.

In Figure 1, the horizontal axis measures the rate of accumulation and the vertical axis measures the rate of investment.  $OP$  is the Penrose curve that expresses the relationship between the rate of investment and capital accumulation. There exists point  $A(\delta + \gamma, r_\alpha(t))$  through which we may draw straight line tangents to the Penrose curve at

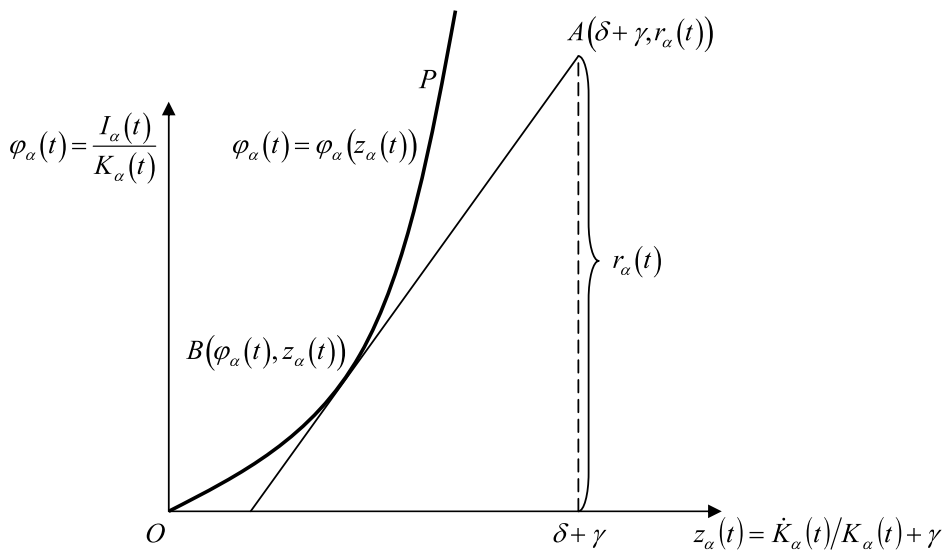


Figure 1 Optimal Accumulation of Culture Capital

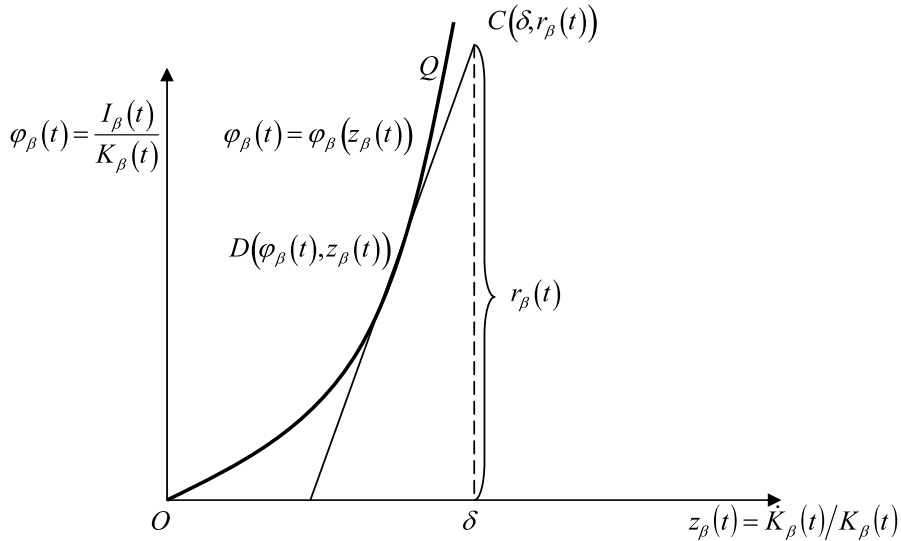


Figure 2 Optimal Accumulation of Productive Capital

point  $B(\varphi_\alpha(t), z_\alpha(t))$  where  $\varphi_\alpha(t)$  is the rate of investment and  $z_\alpha(t)$  is the rate of accumulation. Optimal rates of investment and accumulation are uniquely determined by the sum of marginal rate of substitution between capital endowed in culture and consumption over all households. It is easily seen from Figure 1 that at given discount rate  $\delta$  and depreciate rate  $\gamma$ , the higher sum of marginal rate of substitution between cultural capital and consumption,  $r_\alpha(t)$ , the higher is the rate of accumulation of cultural capital.

We can draw similar figure for productive capital. In Figure 2,  $OQ$  is the Penrose curve. There exists point  $C(\delta, r_\beta(t))$  through which we may draw a straight line tangents to the Penrose curve at point  $D(\varphi_\beta(t), z_\beta(t))$ . Optimal rate of accumulation of capital is uniquely determined by marginal productivity of productive capital. At given discount rate  $\delta$ , the higher marginal productivity of productive capital,  $r_\beta(t)$ , the higher is the rate of accumulation of productive capital.

In our study point A differs with point C along horizontal axis at depreciation rate of cultural capital,  $\gamma$ , which may be interpreted that particular attention is paid to the depreciation of

cultural capital by current generation. Care of maintenance of cultural capital at current state is taken by current generations. People in future generation may benefit from the prevention activity by the current generation.

### 3.2 Optimal Capital Accumulation with Existence of Externality

It is possible to suggest that cultural capital may play an important role in the production. If the cultural factors are conducive to more effective decision making, to more rapid and varied innovation and to more adaptive behavior in dealing with change, the economic productivity and dynamism of the group will be likely eventually to be reflected in better outcomes.

On the other hand, economic activity may also affect the birth of cultural value. High rapid urbanization and industrial development may generate values or standardized form of mass culture; it can be seen as a potential threat to the cultural capital, perhaps leading to the distortion of its intangible cultural value.

The mutual influence between cultural

capital and economic activity may be viewed as the existence of externality, which represents the connection between the sphere of culture and economy lying outside the price system. In this section we will present the dynamic optimum accumulation of capital with the existence of externality. The externality effect in

$$G(K_\alpha(t), f(K_\beta(t)))$$

and

$$f(G(K_\alpha(t)), K_\beta(t))$$

are respectively generated by the productive activity and cultural capital. We assume that externality of cultural capital in the procession of production is positive and the externality of productive activity in the generation of cultural value is negative, given by

$$f_{G(t)}' > 0$$

and

$$G_{f(t)}' < 0.$$

Solving the maximization problem expressed by the utility integral subject to the feasibility constraint, we get the conditions for the imputed price of cultural and productive capital:

$$\frac{\dot{\lambda}_2(t)}{\lambda_2(t)} = \delta + \gamma - z_\alpha(t) - \frac{s_\alpha(t) - \varphi_\alpha(z_\alpha(t))}{\frac{\partial \varphi_\alpha(t)}{\partial z_\alpha(t)}} \quad (38)$$

$$\frac{\dot{\lambda}_3(t)}{\lambda_3(t)} = \delta - z_\beta(t) - \frac{s_\beta(t) - \varphi_\beta(z_\beta(t))}{\frac{\partial \varphi_\beta(t)}{\partial z_\beta(t)}} \quad (39)$$

$s_\alpha(t)$  in (38) is defined as the sum of rate of marginal substitution cultural capital and private goods consumption over all households and the marginal productivity of cultural capital in the process of production, given by

$$s_\alpha(t) = \frac{\sum_{h=1}^H \frac{\partial u^h(t)}{\partial G(t)} \frac{\partial G(t)}{\partial K_\alpha(t)}}{\frac{\partial u^h(t)}{\partial c^h(t)}} + \frac{\partial f(t)}{\partial G(t)} \frac{\partial G(t)}{\partial K_\alpha(t)}$$

and  $s_\beta(t)$  in (39) is defined as the sum of marginal rate of substitution between productive capital and private goods consumption and marginal productivity of productive capital, given by

$$s_\beta(t) = \frac{\sum_{h=1}^H \frac{\partial u^h(t)}{\partial G(t)} \frac{\partial G(t)}{\partial f(t)} \frac{\partial f(t)}{\partial K_\beta(t)}}{\frac{\partial u^h(t)}{\partial c^h(t)}} + \frac{\partial f(t)}{\partial K_\beta(t)}$$

Since marginal rate of substitution may be interpreted as valuation,

$$\sum_{h=1}^H \frac{\frac{\partial u^h(t)}{\partial G(t)} \frac{\partial G(t)}{\partial f(t)} \frac{\partial f(t)}{\partial K_\beta(t)}}{\frac{\partial u^h(t)}{\partial c^h(t)}} < 0$$

in  $s_\beta(t)$  may be understood as social valuation on productive capital from cultural point of view.

Putting  $\dot{\lambda}_2(t)/\lambda_2(t)$  and  $\dot{\lambda}_3(t)/\lambda_3(t)$  to be zero, the equation (38) and (39) can be respectively written as

$$\frac{\partial \varphi_\alpha(t)}{\partial z_\alpha(t)} = \frac{s_\alpha(t) - \varphi_\alpha(z_\alpha(t))}{\delta + \gamma - z_\alpha(t)}$$

and

$$\frac{\partial \varphi_\beta(t)}{\partial z_\beta(t)} = \frac{s_\beta(t) - \varphi_\beta(z_\beta(t))}{\delta - z_\beta(t)}$$

The determination of rate of accumulation,  $z_\alpha(t)$  and  $z_\beta(t)$  may be respectively illustrated by Figure 3 and Figure 4 in comparison to the case without externality.

It is easily seen in Figure 3 that the straight line through  $E(\delta + \gamma, s_\alpha(t))$  is drawn tangent to the Penrose curve  $OP$  at point  $F$  where

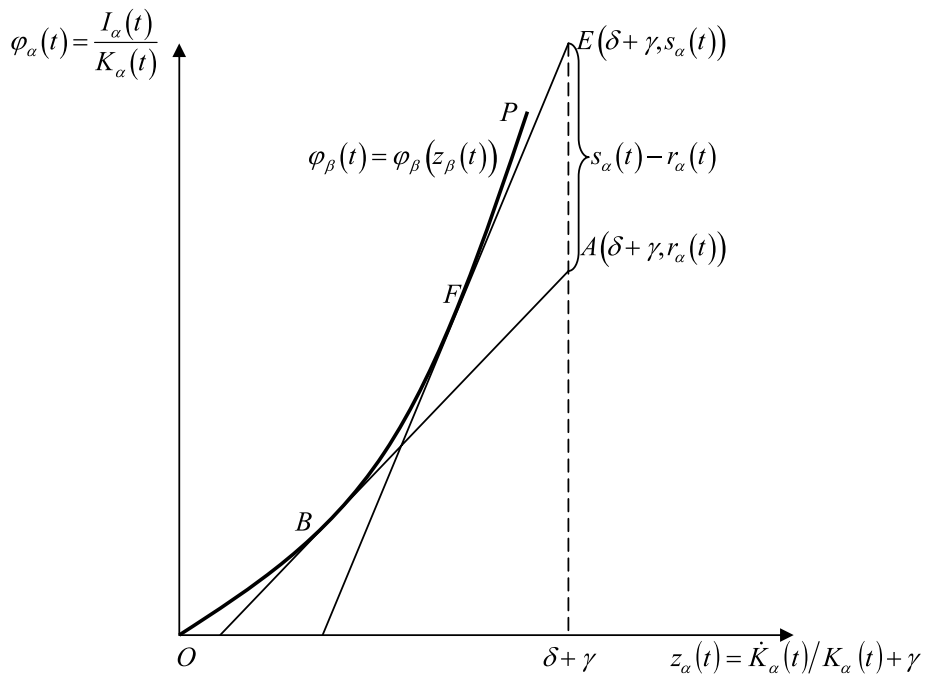


Figure 3 Optimal Accumulation of Culture Capital with Existence of Externality

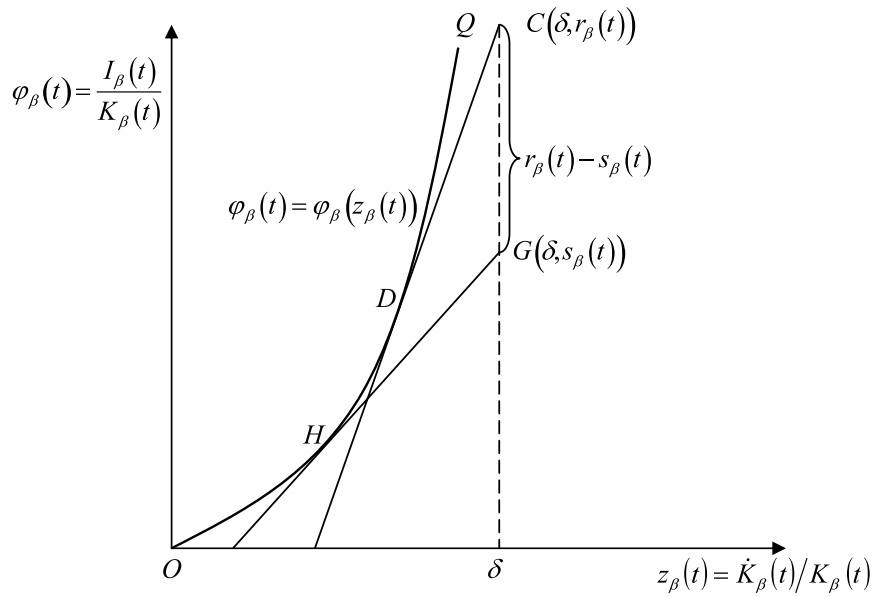


Figure 4 Optimal Accumulation of Productive Capital with Existence of Externality

the rate of accumulation of cultural capital is uniquely determined. At given discount rate  $\delta$  and depreciate rate  $\gamma$ , the higher  $s_\alpha(t)$ , the higher is the rate of accumulation of cultural capital. Point  $E$  is  $s_\alpha(t) - r_\alpha(t)$ <sup>8</sup> higher to point  $A$ , which results the rate of accumulation of cultural capital denoted by point  $F$  is higher than that without externality.

In Figure 4, the straight line through  $G(\delta, s_\beta(t))$  is drawn tangent to the Penrose curve  $OQ$  at point  $H$  where the rate of accumulation of productive capital is uniquely determined. At given discount rate  $\delta$ , the higher  $s_\beta(t)$ , the higher is the rate of accumulation of productive capital. Point  $G$  is  $r_\beta(t) - s_\beta(t)$ <sup>9</sup> lower to point  $C$ , which results the rate of accumulation of productive capital denoted by point  $H$  is lower than that without externality.

Our comment on the results in this section may be short but definite. The mutual influence between culture and economy must be taken serious consideration. Economic growth must give a way to culture development at some degree as long as there exists external economy of culture and external diseconomy of production.

#### 4. Conclusion

We have presented Pareto efficient allocation of the scarce resources in both cultural and productive use from the social planner's point of view. In the standard neoclassical model of the economy, market exists to enable mutually beneficial exchange to occur, and according to the theory of general equilibrium such market will lead to maximization of welfare. Since for social, institutional and technological reasons, cultural capital cannot be privately appreciated, collective ownership of cultural capital may occur. Our study has shown that market failure does exist in

the allocation of cultural capital.

The cause of this inefficient result is that each unit of capital invested in production is paid at real interest rate, but capital endowed in culture is paid at zero. Moreover, cultural benefit arisen from individual capital contribution cannot be exclusively appropriated by his/her own. Hence, the free rider problem arises when individual household realizes that he/she can share the cultural benefit from others' contribution without being charged. Individual household may have an incentive to enjoy the benefits from cultural capital provided by others while providing it insufficiency.

Since private contribution to cultural capital in decentralized market is scarce because of its low return, a tax-subsidy mechanism has been introduced to achieve Pareto efficient equilibrium. Its policy implication lies in that in order to correct households' behavior towards an appropriate provision of cultural capital, each unit of capital endowed in culture should be subsidized at its opportunity cost (the real interest rate), on the other hand, this subsidy should be financed by the collection of a set of personalized tax (individual valuation on cultural capital).

Unfortunately, in our study an individual household is facing the situation that his/her truly expressed valuation on cultural capital will become the price or tax he/she has to pay. Hence, there may be no incentive for an individual household to announce his/her preference at real level. On the other hand, the government may lack knowledge of households' preference and may tend to deal diversified preference with uniform tax system. For the reasons mentioned above, there is room for further study.

Additionally, we have presented the sustainable accumulation process of cultural capital from the viewpoint of the intergenerational distribution of utility. Optimal cultural

<sup>8</sup>  $s_\alpha(t) - r_\alpha(t)$  is  $\frac{\partial f(t)}{\partial G(t)} \frac{\partial G(t)}{\partial K_\alpha(t)}$ , denotes the marginal productivity of cultural capital in the process of production, which is assumed to be positive.

<sup>9</sup>  $s_\beta(t) - r_\beta(t)$  is  $\sum_{h=1}^H \frac{\partial u^h(t) \partial G(t)}{\partial G(t) \partial f(t)} \frac{\partial f(t)}{\partial K_\beta(t)} / \frac{\partial u^h(t)}{\partial c^h(t)}$ , denotes social valuation on the productive capital from the view point of culture, which is assumed to be negative.

development and economic growth are determined respectively by social valuation on cultural capital and marginal productivity of productive capital.

More and more phenomena in modern society support our assumption that cultural and economic behavior may have mutual influence. In what follows, two indicators that link the spheres of culture and economy come into the world, which are cultural capital's marginal productivity in the process of production and social valuation of productive activities from cultural point of view. The result of our study indicates that once cultural capital plays a role as a positive factor in the process of production and economic activities may have negative effects on culture change, the economic growth must give a way to cultural development at some degree.

This result leads to an immediate policy implication that the government should establish policies which recognize the pervasive importance of culture in development. In the no-policy market economy, individuals tend to ignore the positive external effects of cultural capital. Consequently, less cultural capital stock will be accumulated. The decision problem is to choose point  $F$  in Figure 3, such that the cultural sustainability condition holds. In other words, in any given period society would need to allocate a sufficient level of resources to utilize its cultural capital stock and would need to re-invest a sufficient level of the financial income in the conservation and maintenance of the stock, in order to ensure no deterioration in the cultural value of the stock in the next period.

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