Rental Market and Quality Improvement*

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1. Introduction

We say that durable goods are of a good quality when they not only yield large service streams but also exhibit considerable durability. These two measures of quality share a unique relationship. The perceived value of durability is lowered when new models yielding larger service streams are introduced : previously manufactured durable goods come to be obsolete from an economic standpoint by the introduction of new models yielding larger service streams, and therefore, whether or not durable goods have considerable durability is less important if new models are introduced[1].

There is extensive literature concerning the durability of goods. In a series of seminal papers, Swan (1970) argued that both monopolist and competitive firms choose socially optimal durability levels that minimize the cost of producing service streams. On the other hand, Coase (1972) and Bulow (1986) pointed out that monopolists have an incentive to reduce durability because of time consistent problems. However, these studies do not consider environments where goods of better quality are introduced repeatedly. Lee and Lee (1997) and Waldman (1993, 1996) focused on the obsolescence of

1) To avoid confusion, we do not use the word "quality" to express durability from now on.

goods by technological development. More specifically, they focused on the conduct of a monopolist given the durability of goods but did not study effects of the choice of durability on social welfare.

This paper proposes a model in which homogeneous consumers purchase goods repeatedly assuming durability is determined endogenously while technological development is given exogenously. This approach enables us to analyze the relationship between the socially optimal durability level and the economical obsolescence of goods. We have found that social welfare under a competitive rental market is smaller than under a seller's market if new and improved models are introduced repeatedly.

2. The Model and the Social Optimum

The horizon is infinite and time is continuous. The producer has technologies to produce goods of quality Q_t at any time. These technologies progress with time at a constant rate a : $Q_t = A_0 + at$ assuming learning by doing, spillover from other industries, and so on. Although constant returns to scale technology is assumed, the marginal cost of production is a function of durability : $C(d)$ where d denotes the durability of a good, that is, the time until breakdown from the date of production. It is also assumed that $C'(d) > 0$, $C''(d) > 0$, $C(0) > 0$[2].

There is a continuum of homogeneous consumers of measure 1, and each can consume either one or zero durable good at each instance. A consumer that possesses a durable good produced at date t obtains utility flow $u = Q_t + I$, where I is the net income. The instantaneous interest rate r $(0 < r < 1)$ is constant and is the same for consumers and producers.

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2) The assumption $C(0) > 0$ is justified if we agree that a television with a minute durability cannot be produced with close to zero cost.

Under constant returns to scale, a social planner distributes goods to all consumers after he/she has decided on some production date. Let positive t_n $(n=1, 2, \ldots)$ denote the date at which the goods are produced. Note that the durability of generation n is chosen to be equal to $t_{n+1} - t_n$ since redundant durability alone is useless[3] . The social planner maximizes the following expression :

$$SW = \int_{t_1}^{t_2} e^{-rs}(A_0 + at_1)\,ds - C(t_2 - t_1) + \cdots$$

$$+ \int_{t_n}^{t_{n+1}} e^{-rs}(A_0 + at_n)\,ds - e^{-rt_n}C(t_{n+1} - t_n) + \cdots$$

This is the discounted present value of the consumer surplus minus production costs. The first-order conditions for this problem are, for $n=1$,

$$-e^{-rt_1}(A_0 + at_1) + \frac{e^{-rt_1} - e^{-rt_2}}{r}a + re^{-rt_1}C(t_2 - t_1)$$

$$+ e^{-rt_1}C'(t_2 - t_1) = 0 \qquad\qquad (1)$$

and, for $n=2, 3, \ldots$,

$$-e^{-rt_n}a(t_n - t_{n-1}) - e^{-rt_{n-1}}C'(t_n - t_{n-1}) + \frac{e^{-rt_n} - e^{-rt_{n+1}}}{r}a$$

$$+ re^{-rt_n}C(t_{n+1} - t_n) + e^{-rt_n}C'(t_{n+1} - t_n) = 0$$

In a steady state where $t_{n+1} - t_n = d \quad \forall n$[4],

$$-a\left(d - \frac{1 - e^{-rd}}{r}\right) - (e^{rd} - 1)C'(d) + rC(d) = 0 \qquad\qquad (2)$$

Note that if initial quality A_0 is so high that it compensates for the production cost and the quality of the goods is not improved, $a=0$, the first-order condition is,

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3)　This is because there is no second-hand market. We cannot analyze a second-hand market since homogenous consumers are assumed.

4)　If initial quality is so high that the first models are introduced at $t=0$, all intervals of production are constant. See Appendix A.

$$\frac{C'(d)}{C(d)} = \frac{r}{e^{rd}-1} \tag{3}$$

This condition is the same as that of Swan (1970), who did not consider quality improvements. Therefore, the first term of (2) reflects an effect of quality improvement.

The following proposition shows the effect that the speed of quality improvement has on the optimal production plan.

Proposition 1

The socially optimal durability level decreases with a.

Proof

Totally differentiating (2) gives,

$$\frac{\partial d}{\partial a} = \left(\frac{1-e^{-rd}}{r}d\right) \Big/ (1-e^{-ed})\left(a+\frac{C''(d)+rC'(d)}{e^{-rd}}\right).$$

The denominator is clearly positive while the numerator is negative since

$$1-e^{-rd} = \int_0^d e^{-rs}ds < \int_0^d ds = d.$$

Q. E. D.

Shortening the interval of production enables us to benefit from technological developments effectively while the total production cost increases. When the speed of improvement is high, because the former effect is more important than the latter, the optimal interval (durability) is relatively short.

3. Market Equilibrium

We consider two types of duopoly markets : a seller's market and a rental market. All models are introduced by two infinitely living firms that control prices (rental prices) and production schedules to maximize profits. It is assumed that one of the firms can capture the entire market if it gives consumers a larger surplus than its competitor. It is also assumed that each firm

can obtain one half of the total demand by adopting the same policy.

Since the horizon is infinite, there can be a countless equilibrium if the strategy is unrestricted. To reduce the equilibrium set, we consider only (i) a pure-strategy equilibrium and (ii) a strategy that depends only on data that directly affects profits. For simplicity, we assume that initial quality A_0 is sufficiently large so that producing goods at $t_1=0$ is profitable.

Seller's Market

Suppose that firm 1 introduces goods with a durability satisfying (2) and sells the goods at a price equal to marginal cost. If firm 2 adopts a strategy that differs from that of firm 1, it cannot capture any demand at all, with consumers enjoying the maximum surplus by buying the durable goods from firm 1 repeatedly. Therefore, on the equilibrium, both firms introduce goods with a durability satisfying (2) at a price equal to marginal cost : the social welfare is maximized in the competitive seller's market.

Rental Market

In a competitive rental market, consumers can choose the firm they want to rent goods from at every instance : from the firm supplying the largest net service stream. Note that these consumers have no interest in the durability of the rented goods.

Let us assume that the firms must commit to a constant rental price until the next production date[5]. Let $a^t=$(N, P with p_r) denote a firm's action at date t where "N" means that new generation models are not produced while "P with p_r," means that the firm produces new generation models and its rental price is p_r.

Consider the strategy of

5) If firms can revise rental prices freely, the rental market will not operate. The reason is that competition makes the equilibrium price equal to zero, since production costs are sunk once goods are produced.

$$a^t = \text{P with } p^r \quad \text{if} \quad t = id^*$$
$$= N \quad \text{if} \quad t \neq id^*$$

where $i = 0, 1, 2, \ldots, d^*$ satisfies (2) and $p^r = \dfrac{r}{1 - e^{-rd^*}} C(d^*)$[6]. This strategy does not constitute equilibrium since there is incentive for a firm to deviate. One of the firms can preempt the entire demand by supplying a larger net service stream than its competitor. There are two possible ways: the firm can (i) charge a lower rental price since the durability level satisfying (2) is not cost minimization or can (ii) rent goods with a better quality by delaying production since the quality of goods produced at later dates is better than the quality of goods produced at earlier dates[7].

The strategy that maximizes the net service stream can be expected to constitute equilibrium. Given that the next generation goods are produced at time t_{n+1}, the production time of the generation n is given by

$$\max_{t_n} \ A_0 + at_n - \frac{r}{1 - e^{-r(t_{n+1} - t_n)}} C(t_{n+1} - t_n) \tag{4}$$

By writing $t_{n+1} - t_n = d$, the first-order condition is,

$$-\frac{(1 - e^{-rd})^2}{re^{-rd}} a - (e^{rd} - 1) C'(d) + rC(d) = 0. \tag{5}$$

Let d^r be the solution of (5).

Lemma 1

In a rental market, the only strategy that constitutes a sub-game perfect equilibrium is,

$$a^t = \text{P with } p^r \quad \text{if} \quad t = id^r$$
$$= N \quad \text{if} \quad t \neq id^r \tag{6}$$

where $i = 0, 1, 2, \ldots$, and $p^r = \dfrac{r}{1 - e^{-rd^r}} C(d^r)$.

Proof

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6) This rental price is given by the zero profit condition: $\int_0^{d^*} e^{-rd^*} p_r ds = C(d^*)$.

7) In this model, the cost minimizing durability is given by (3).

See Appendix B.

Since the maximization of problem （4） does not consider the dynamic efficiency, d^r is not the optimum durability in terms of social welfare except that （2） coincides with （5） by chance. Finally, we obtain the following proposition.

Proposition 2

Social welfare under a rental market is generically smaller than under a seller's market.

4. Concluding Remarks

We have shown the optimal durability level and explained that social welfare is not maximized in a competitive rental market when there are improvements in quality. Further research focusing on the market structure, the case of heterogeneous consumers, and so on, might be interesting.

Appendix A

Following argument is due to Fishman and Rob (2000).

Assume that the goods are produced at some t. Let $V(t)$ denote the value function for the planning problem right after the introduction of this product. Then V satisfies the following Bellman equation,

$$V(t) = \max_d \left\{ \frac{1-e^{-rd}}{r}(A_0 + at) - C(d) + e^{-rd}V(t+d) \right\}.$$

Subtracting $\dfrac{A_0 + at}{r}$ from both sides and defining $\widetilde{V}(t) - \dfrac{A_0 + at}{r}$, we get

$$\widetilde{V}(t) = \max_d \left\{ \left[\frac{e^{-rd}}{r}ad - C(d) \right] + e^{-rd}\widetilde{V}(t+d) \right\}.$$

Since the term in the square bracket does not depend on t, the new value

function, $\widetilde{V}(t)$ is also independent of t. It then follows that for given d,

$$\widetilde{V}(t) = \frac{1}{1-e^{-rd}}\left\{\frac{e^{-rd}}{r}ad - C(d)\right\}$$

and therefore

$$V(t) = \frac{A_0 + at}{r} + \frac{1}{1-e^{-rd}}\left\{\frac{e^{-rd}}{r}ad - C(d)\right\}.$$

When current state is $t=0$,

we can express objective function by

$$V = \left\{\frac{A_0}{r} + \frac{1}{1-e^{-rd}}\left(\frac{e^{-rd}}{r}ad - C(d)\right)\right\}.$$

Regardless to say, differentiating this with respect to d gives the equation (2).

Appendix B

Proof of Lemma 1

　　We show that strategy (6) constitutes a sub-game perfect equilibrium. On the equilibrium, firms cannot adopt different strategies, since one of the firms can make non-negative profits by producing at the same time as its competitor. Therefore, we have only to show that both firms do not have any incentive to deviate from strategy (6).

　　First, suppose that firm 1 adopts strategy (6) while firm 2 deviates and produces goods at $t_d \in [id^r, (i+1)d^r]$, which eventually break down at $t_d + k \in [jd^r, (j+1d^r)]$ (where $i < j$ is an integer and k is the durability) and rents the goods at the lowest possible price to compensate for production costs:

$$p^r = \frac{r}{1-e^{-rk}}C(k).$$

　　To show that firm 2 cannot obtain demand after jd^r, let us compare the net service stream supplied by firm 1 to firm 2 at time jd^r. For this purpose,
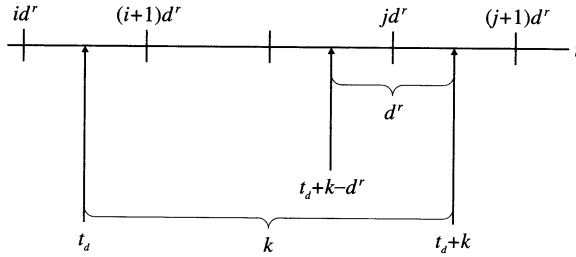
Figure 1

it is useful to consider the auxiliary case of producing the goods with durability d^r at time $t_d + k - d^r$. See **Figure 1**. Noting that d^r is the solution of (5) and regarding $t_d + k$ as t_{n+1}, we obtain

$$A_0 + at_d - \frac{r}{1-e^{-rk}}C(k) = A_0 + a(t_d + k - k) - \frac{r}{1-e^{-rk}}C(k)$$

$$< A_0 + a(t_d + k - d^r) - \frac{r}{1-e^{-rd^r}}C(d^r)$$

and comparing the net service stream provided by firm 1 to the auxiliary case,

$$A_0 + a(t_d + k - d^r) - \frac{r}{1-e^{-rd^r}}C(d^r)$$

$$< A_0 + a((j+1)d^r - d^r) - \frac{r}{1-e^{-rd^r}}C(d^r) = A_0 + ajd^r - \frac{r}{1-e^{-rd^r}}C(d^r)$$

These inequalities show that the net service stream supplied by firm 1 is larger than that supplied by firm 2 at time jd^r, and therefore, firm 2 cannot obtain any further demand after time jd^r. Needless to say, firm 2 cannot obtain any demand if it rents goods at a higher price. Since redundant durability only lowers firm 2's profit, the firm cannot adopt this plan.

Next, consider the plan of producing goods at time td, which eventually break down at time jd^r, and renting them at $p^r = \dfrac{r}{1-e^{-r(jd^r-k)}}C(jd^r - t_d)$. Noting that firm 1 produces goods with durability d^r at time $(j-1)d^r$, firm 2 cannot obtain any demand after $(j-1)d^r$ since,

$$A_0 + at_d - \frac{r}{1 - e^{-r(jd^r - t_d)}} C(jd_r - t_d) < A_0 + a(j-1)d^r - \frac{r}{1 - e^{-rd^r}} C(d^r).$$

Applying this procedure repeatedly, we can confirm that a deviation plan does not exist by which firm 2 can obtain demand.

On the other hand, it is apparent that strategy (6) only constitutes a sub-game perfect equilibrium. If firm 2 adopts a strategy other than (6), firm 1 can always preempt the demand by adopting strategy (6) since the quality of its goods is better or it can choose to charge a lower rental price than its competitor or both.

Q. E. D

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