# On the Optimal Tax Schemes, Expenditure Plans and the Environmental Externality

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## 1 Introduction

Ono (1996) analyzes the optimal tax schemes in an overlapping generations model with environmental externality. Using the framework of John and Pecchenino (1995), he analizes two sorts of consumption tax schemes to achieve a socially optimal allocation.

In this paper, we reexamine Ono's analysis of the case of a consumption tax accompanied by an interest income tax by clarifying the effects of the government expenditure plan. Three types of government expenditure plans are considered: transfer payments, government consumption, and government investment. We show that whether the tax schemes can successfully achieve socially optimal allocation depends on how the government spends the tax revenue. In section 2, we explain the model. We investigate the optimal tax schemes and the government expenditure plan in section 3. A summary is in section 4.

## 2 The Model

In this section, we explain the economic environment. Then we derive the conditions for a socially optimal allocation and a competitive allocation of this economy.

## 2.1 The Economic Environment

We consider an overlapping generations model with no population growth[1]. The economy and nature have infinite-horizons but agents have a finite life time. At each period a new generation is born and lives two periods. A representative agent of generation born at period $t$ has a utility function

$$u(c_t^1, c_{t+1}^2, E_{t+1}), \tag{1}$$

where $c_t^1$ and $c_{t+1}^2$ are the agent's consumption levels at period $t$ and $t+1$ respectively. $E_{t+1}$ is an index of environmental quality at period $t+1$. Agents supply their endowed labor forces to firms inelastically and earn wage income when they are young. They consume some part of their wage income, pay for the maintenance of the environment and save for consumption during the second period of their lives. When they are old, they consume all their savings and leave no bequest.

Firms maximize their profits under perfect competition with the standard neoclassical production function $y_t = f(k_t)$, where $y_t$ and $k_t$ are per capita output and capital stock at period $t$. From profit maximization, the following conditions hold in a competitive equilibrium.

$$w_t = f(k_t) - k_{t+1} f'(k_t),$$

$$r_{t+1} = f'(k_{t+1}) - \delta,$$

where $\delta \in (0, 1)$ is the capital depreciation rate.

The environmental quality index of period $t+1$ is determined by

$$E_{t+1} = (1-b)E_t - \beta(c_t^1 + c_t^2) + \gamma m_t, \tag{2}$$

1) The economic environment is exactly same as Ono (1995). Yoshida (1999) investigates the pigouvian tax systems in an overlapping generation model with elastic labor supply. He considers a wage tax and a consumption tax in two cases : when the environment is a public consumption good and when it is a public intermediate good.

where $b \in (0, 1)$, $\beta > 0$ and $\gamma > 0$. The parameter $b$ measures the speed of autonomous revolution of environment quality. The term $\beta(c_t^1 + c_t^2)$ represents the degradation of the environment caused by consumption at period $t$, and $\gamma m_t$ represents environmental improvement from maintenance activity $m_t$[2)].

## 2.2 Socially Optimal Allocation

A social planner treats all generations the same and chooses $c^1$, $c^2$, $m$, $k$, to maximize the utility of the representative generation subject to feasibility and environmental conditions. The social planner's problem is to

$$\max u(c^1, c^2, E)$$

subject to

$$f(k) + (1-\delta)k = c^1 + c^2 + k + m,$$

$$E = -\frac{\beta}{b}(c^1 + c^2) + \frac{\gamma}{b}m.$$

Socially optimal allocation in a steady state $(\bar{c}^1, \bar{c}^2, \bar{m}, \bar{k}, \bar{E})$ is defined by the following conditions.

$$u_1 = \frac{\beta + \gamma}{b}u_3, \tag{3}$$

$$u_2 = \frac{\beta + \gamma}{b}u_3, \tag{4}$$

$$f'(\bar{k}) = \delta, \tag{5}$$

$$f(\bar{k}) + (1+\delta)\bar{k} = \bar{c}^1 + \bar{c}^2 + \bar{k} + \bar{m} \tag{6}$$

$$\bar{E} = -\frac{\beta}{b}(\bar{c}^1 + \bar{c}^2) + \frac{\gamma}{b}\bar{m} \tag{7}$$

## 2.3 Competitive Equilibrium allocation

Let's derive the conditions for a competitive equilibrium and verify that the competitive equilibrium allocation is different from the socially optimal

2) E can be thought of as an index of the natural capital stock, b as the natural depreciation rate of natural capital, and $\gamma m$ as investment in natural capital.

allocation.

The representative agent of the generation $t$ treats $w_t$, $r_{t+1}$, $E_t$, $c_t^2$ as givens and maximizes the utility function (1) with respect to $c_t^1$, $c_{t+1}^2$, $s_t$, $m_t$ subject to[3)]

$$c_t^1 + s_t + m_t = w_t,$$

$$c_{t+1}^2 = (1 + r_{t+1}) s_t,$$

$$E_{t+1} = (1-b) E_t - \beta (c_t^1 + c_t^2) + \gamma m_t,$$

The competitive allocation in a steady state $(c^1, c^2, m, k, E)$ is characterized by the following conditions.

$$u_1 = (\beta + \gamma) u_3, \tag{8}$$

$$u_2 = \frac{\gamma}{1 - \delta + f'(k)} u_3, \tag{9}$$

$$c^1 + k + m = f(k) - kf'(k),$$

$$c^2 = \{1 - \delta + f'(k)\} k,$$

$$E = -\frac{\beta}{b}(c^1 + c^2) + \frac{\gamma}{b} m.$$

Comparing (8) and (9) with (3) and (4), we know that the intergenerational long-run effect $1/b$ does not appear in either (8) or (9). Agents do not care about the quality of the environment after their life time. Therefore, an intergenerational long-run effect of the environmental externality $1/b$ is not internalized in a competitive equilibrium. Moreover, $\beta$ does not appear in condition (9) because the young generation cares about the degradation effect of consumption on the environmental quality $\beta$, but the old generation does not.

3) The representative agent takes equation (2) as a constraint for the utility maximization problem and chooses the optimal level of $m$. Under this formulation of utility maximization problen, there is an assumption that there exists a one-period lived government which levies the optimal level lump-sum tax $m$ to maximize the utility of generation $t$, or the assupmtion that a social planner sets the optimal price for the quality of environment for generation $t$. See John, and Pecchenino (1994) and John et al. (1995). In stead of this assumption, we may be able to suppose that at each period all young agents organize a NPO to maintain environment quality and voluntarily donate some of their income and leisure time for maintenance activities.

## 3 Optimal Tax Schemes and Government Expenditure Plans

In this section, we introduce a consumption tax $\tau_c$, an interest income tax $\tau_k$ and a lump-sum tax $\tau$ in this economy in the same way as Ono (1996)[4]. A lump-sum tax which is always transfered to the old generation is necessary to obtain a golden rule level of capital stock. To balance the government budget, the government has to spend the tax revenue from both the consumption tax and the interest income tax. We reexamine the optimal tax schemes under several government expenditure plans. First we consider the case in which the government spends all of the tax revenue as a transfer payment. Then we consider the cases of government consumption and government investment in natural capital.

### 3.1 Government Expenditure for Transfer Payments

At each period, the government transfers all current tax revenue from both consumption tax and interest income tax to the agents who live in the period. Let's denote the tax revenue at period t as $v_t$. Suppose that the government transfers the $\alpha_t$ of the $v_t$ to the young and $(1-\alpha_t)$ of $v_t$ to the old. Then the problem which the generation $t$ solves is to maximize (1)

subject to

$$(1+\tau_c)c_t^1+s_t+m_t=w_t-\tau_t+\alpha_t v_t,$$

$$(1+\tau_c)c_{t+1}^2=(1+r_{t+1})(1-\tau_k)s_t+\tau_{t+1}+(1-\alpha_{t+1})v_{t+1},$$

$$E_{t+1}=(1-b)E_t-\beta(c_t^1+c_t^2)+\gamma m_t,$$

where $v_{t+i}=\tau_c c_{t+i}+\tau_k(1+r_{t+i})s_{t+i-1}$, $\alpha_{t+i}\in[0,\ 1]$ and $c_{t+i}=c_{t+i}^1+c_{t+i}^2$, $i=0,\ 1$.

4) Ono (1996) also discusses the optimal tax scheme when government can differentiate the consumption tax rates between young and old. In that case, we are able to discuss similarly as this section.

The competitive allocation ($c^1$, $c^2$, $m$, $k$, $E$) in a steady state equilibrium is characterized by

$$u_1=[\beta+\{1+(1-\alpha)\tau_c\}\gamma]u_3, \tag{10}$$

$$u_2=\frac{(1-\alpha\tau_c)\gamma}{\{1-\delta+f'(k)\}(1-\alpha\tau_k)}u_3, \tag{11}$$

$$(1+\tau_c)c^1+k+m=f(k)-kf'(k)-\tau+\alpha[\tau_c c+\tau_k\{1-\delta+f'(k)\}k],$$

$$(1+\tau_c)c^2=\{1-\delta+f'(k)\}k+\tau_c c+\tau+(1-\alpha)[\tau_c c+\tau_k\{1-\delta+f'(k)\}k],$$

$$E=-\frac{\beta}{b}c+\frac{\gamma}{b}m.$$

Let's consider a transfer plan that the government transfers all of the tax revenue to the old generation as well as Ono's. In this case, $\alpha_t=\alpha_{t+1}=0$, $\forall t$. Then the conditions (10) and (11) are

$$u_1=\{\beta+(1+\tau_c)\gamma\}u_3, \tag{12}$$

$$u_2=\frac{\gamma}{1-\delta+f'(k)}u_3. \tag{13}$$

Then by setting

$$\tau_c=\frac{(1-b)(\beta+\gamma)}{b\gamma},$$

the condition ( 3 ) holds. However, we cannot set the tax rates to hold condition ( 4 ). As we see from (13), neither $\tau_c$ nor $\tau_k$ can affect the optimal condition for consumption by the old generation because agents take into account that the tax they pay at old age returns to themselves immediately. Because of this we cannot find the optimal tax scheme which achieves a socially optimal allocation in this expenditure plan[5)].

Next, suppose that the government transfers the all tax revenue from both consumption tax and interest income tax to the young generation. In this case $\alpha_t=\alpha_{t+1}=1$, $\forall t$. The conditions (10) and (11) become

$$u_1=(\beta+\gamma)u_3,$$

5) Therefore, we cannot prove Ono's 2nd proposition which is "The optinal allocation can be achieved by a consumption tax, an interest income tax, and a lump-sun tax on the young".

$$u_2=\frac{(1-\tau_c)\gamma}{\{1-\delta+f'(k)\}(1-\tau_k)}u_3.$$

No optimal tax scheme exists in this transfer plan also. Therefore, to obtain the optimal tax scheme, $\alpha_t \in (0, 1)$ must hold if the government transfers the tax revenue.

Lets consider next a transfer plan in which the government transfers the revenue from the interest income tax to the young generation and transfers the revenue from the consumption tax to the old generation[6]. That is

$$\alpha_{t+i}=\frac{\tau_k(1+r_{t+i})s_{t+i-1}}{\tau_c c_{t+i}+\tau_k(1+r_{t+i})s_{t+i-1}}, \quad i=0,\ 1 \quad \forall t.$$

In this case, the conditions (10) and (11) become

$$u_1=\{\beta+(1+\tau_c)\gamma\}u_3, \tag{14}$$

$$u_2=\frac{\gamma}{\{1-\delta+f'(k)\}(1-\tau_k)}u_3. \tag{15}$$

By setting the tax scheme as

$$\tau_c=\frac{(1-b)(\beta+\gamma)}{b\gamma}, \tag{16}$$

$$\tau_k=\frac{\beta+(1-b)\gamma}{\beta+\gamma}, \tag{17}$$

$$\tau=f(\bar{k})-\delta\bar{k}+(1+\delta)\frac{\beta+(1-b)\gamma}{\beta+\gamma}\bar{k}-\frac{(1-b)\beta+\gamma}{b\gamma}\bar{c}^1-\bar{k}-\bar{m}, \tag{18}$$

we can derive conditions identical to the socially optimal conditions ( 3 ) ~ ( 7 ). Now we have shown that Ono's 2nd proposition is valid though this optimal tax scheme is different from Ono's tax scheme.

### 3.2 Government Expenditures for Consumption and Investment

In section 3.1 we considered the case in which the government spends the

6) Obviously, if we set $\alpha$ equal to the share of consumption tax revenue to total tax revenue, the tax scheme has no effect on the economy.

tax revenue for transfer payments. However, the government could also spend the tax revenue for consumption or investment for maintenance of the environment.

Let's consider the case of government consumption first. Suppose that the government consumes all of the tax revenue except the revenue from lump-sum taxes.

Then generation $t$ maximizes (1)
subject to

$$(1+\tau_c)c_t^1+s_t+m_t=w_t-\tau_t,$$

$$(1+\tau_c)c_{t+1}^2=(1+r_{t+1})(1-\tau_k)s_t+\tau_{t+1},$$

$$E_{t+1}=(1-b)E_t-\beta c_t+\gamma m_t.$$

A competitive equilibrium allocation in a steady state is characterized by

$$u_1=\{\beta+(1+\tau_c)\gamma\}u_3,$$

$$u_2=\frac{(1+\tau_c)\gamma}{\{1-\delta+f'(k)\}(1-\tau_k)}u_3,$$

$$(1+\tau_c)c^1+k+m=f(k)-kf'(k)-\tau,$$

$$(1+\tau_c)c^2=\{1-\delta+f'(k)\}(1-\tau_k)k+\tau,$$

$$E=-\frac{\beta}{b}c+\frac{\gamma}{b}m.$$

Then, by setting the tax scheme to

$$\tau_c=\frac{(1-b)(\beta+\gamma)}{b\gamma}>0,$$

$$\tau_k=\frac{b\beta}{\beta+\gamma}>0,$$

$$\tau=f(\bar{k})-\delta\bar{k}-\frac{(1-b)(\beta+\gamma)}{b\gamma}c^1-\bar{k}-m.$$

we derive conditions identical to (3), (4), (5) and (7). This tax scheme is exactly the same as the one derived by Ono and successfully internalizes both the intergenerational externality and the degradation effects of consumption.

However, because the government consumes tax revenue, agents cannot enjoy the socially optimal level of consumption[7].

Let's consider the case of government investment next. Suppose that the government spends the revenue from the interest income tax for the maintenance of the environment and transfers the revenue from the consumption tax to old generation. The problem is to maximize (1)
subject to

$$(1+\tau_c)c_t^1+s_t+m_t=w_t-\tau_t,$$
$$(1+\tau_c)c_{t+1}^2=(1+r_{t+1})(1-\tau_k)s_t+\tau_{t+1}+\mu_{t+1},$$
$$E_{t+1}=(1-b)E_t-\beta c_t+\gamma(m_t+g_t),$$

where $\mu_{t+1}=\tau_c(c_{t+1}^1+c_{t+1}^2)$ and $g_t=\tau_k(1+r_t)s_{t-1}$.

A steady state allocation is characterized by the following conditions.

$$u_1=\{\beta+(1+\tau_c)\gamma\}u_3, \tag{19}$$
$$u_2=\frac{\gamma}{\{1-\delta+f'(k)\}(1-\tau_k)}u_3, \tag{20}$$
$$(1+\tau_c)c^1+k+m=f(k)-kf'(k)-\tau,$$
$$(1+\tau_c)c^2=\{1-\delta+f'(k)\}(1-\tau_k)k+\tau+\tau_c(c^1+c^2),$$
$$E=-\frac{\beta}{b}(c^1+c^2)+\frac{\gamma}{b}(m+g).$$

Notice that (19) and (20) are exactly same as (14) and (15).

Then, let's set the tax scheme to (16), (17) and[8]

$$\tau=f(\bar{k})-\delta\bar{k}-\frac{(1-b)\beta+\gamma}{b\gamma}\bar{c}^1-\bar{k}-\bar{m}. \tag{21}$$

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7) The feasibility condition in this regime is given by
$$f(\bar{k})+(1-\delta)\bar{k}-\{\tau_c(c^1+c^2)+\tau_k\bar{k}\}=c^1+c^2+\bar{k}+m,$$
Therefore, condition (6) is not satisfied. As long as the government consumes any part of tax revenue, a socially optimal allocation defined in section 2.2 cannot be obtained even if the tax scheme suceeds in internalizing the environmental externality.

8) The level of lump-sum tax (21) is lower than (18) because the government does not give a transfer payment to the young generation in this regime.

With this tax scheme, we can derive conditions identical to (3)~(7)[9].

In this model, government expenditure for the maintenance of the environment has the same effect as a government transfer to the young generation because only the young take care of the quality of the environment by paying for maintenance. The government spends tax revenue for maintenance, so the young generation can reduce their expenditures for this maintenance $m$.

## 4 Summary

We have examined the optimal tax schemes with a consumption tax, an interest income tax, and a lump-sum tax by considering how the tax revenues from a consumption tax and an interest income tax are used. A lump-sum tax levied on the young generation must always be transfered to the old generation to obtain a golden rule level of capital stock.

If the government transfers all tax revenue (except the revenue from a lump-sum tax) to a single generation, the tax scheme cannot internalize the intergenerational environmental externality so that an optimal tax scheme does not exist. To create an optimal tax scheme, transfer payments must be diversified across generations. We derive the optimal tax scheme with the transfer policy that the government transfers the interest income tax revenue to the young generation and the consumption tax revenue to the old generation. In this way we confirm Ono's proposition though the optimal tax scheme is different from Ono's.

When the government consumes the tax revenue, the tax scheme which is

9) In this case, condition (6) and (7) hold in the following expressions.

$$f(\bar{k})+(1-\delta)\bar{k}=\bar{c}^1+\bar{c}^2+\bar{k}+(\bar{m}-\tau_k\bar{k}),$$

$$\bar{E}=-\frac{\beta}{b}(\bar{c}^1+\bar{c}^2)+\frac{\gamma}{b}(\bar{m}-\tau_k\bar{k}).$$

derived by Ono internalizes the environmental externality but a socially optimal consumption level cannot be achieved.

Government expenditure for investment in natural capital has the same effect on the economy as a transfer payment to the young generation. Therefore, if the government spends all of the tax revenue for investment in natural capital, an optimal tax scheme does not exist. If the government spend the interest income tax revenue for investment and transfer the consumption tax revenue to the old, a socially optimal allocation can be achieved by such a tax scheme.

We have shown the importance of taking into account the appropriate expenditure plan particuiary which generation benefits from the plan when we plan an optimal tax scheme.

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