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1 Introduction

The cooperative vehicle-pedestrian driving safety support system (CVPDSSS) is an ITS (Intelligent Transport Systems) application belonging to the safety field. This application aims to prevent pedestrian-vehicle accidents by connecting pedestrians and vehicles through signal communications and by alerting drivers of pedestrian presence in their blind spots.

Concerning ITS, systems that mainly aim at resolving traffic congestion, such as ATIS (Advanced Traveler Information Systems) and ETC (Electronic Toll Collection), have already been put to practical use. Many researches have been conducted on the impacts of such systems as well as their market penetration policies. With regard to ATIS, many researchers have actively studied the effect of this system on travel time as well as the impact on driver’s route choice behavior in the 1990s (e.g., Ben-Akiva et al., 1991; Chorus et al., 2009; Emmerink et al., 1995; Levinson, 2003; Yang, 1999). As for the endogenous market penetration model for ATIS, several models have been established (e.g., Emmerink et al., 1996, 1998; Yang, 1998, 1999; Yang and Meng, 2001; Yin and Yang, 2003; Zang and Verhoef, 2005). With regard to ETC, Levinson and Chang (2003) and Fukuda et al. (2004) have established a model explicitly considering the

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network externality of ETC and they have analyzed, from an economic welfare perspective, the impacts of governmental policies for ETC promotion, such as a toll discounts.

In contrast, the CVPDSSS is a system that is currently being developed, and studies regarding its effect and promotion policy have not been conducted thus far. However, based on the following two reasons, we believe that it is important to discuss the promotion policy for this system. The first reason is the serious nature of pedestrian-vehicle accidents. In Japan, among the total number of vehicle accidents that is approximately 665 thousands in the year 2012, the number of pedestrian-vehicle accidents amounts to approximately 64 thousand, that is, slightly lower than 10% of the total number of accidents. However, with regard to the number of fatal accidents, the number of pedestrian-vehicle accidents is 1,567, which accounts for 37% of the total number of fatal accidents (4,233) (National Police Agency, 2013). Therefore, in order to considerably reduce the number of fatal accidents, it is imperative that we decrease the number of pedestrian-vehicle accidents. The second reason is attributed to the CVPDSSS’s feature as a good. In general, goods and services in the IT industry such as the Internet, cell phones, computer software, CDs, DVDs, and video games have a common feature, namely, “network externality” (Katz and Shapiro, 1985, 1994; Libenstein, 1950; Rohlfs, 1974). In other words, the benefit that an individual can enjoy from a good or a service increases as the number of individuals consuming the same good or service augments. Therefore, the expectations of the consumers regarding the penetration level of the concerned good or service will have significant impacts on the actual market penetration level. In comparison, in the case of the CVPDSSS, both the pedestrians and the drivers should possess special devices such as IC-tags and/or short range wireless communication modules to use this system. With regard to the pedestrians, the benefit of possessing the special device (hereinafter, “pedestrian
unit”) depends on the penetration level of the special device for vehicles (hereinafter, “on-board unit”). In contrast, the benefit that drivers can enjoy by possessing the on-board unit depends on the penetration level of the pedestrian unit. Due to this mutual interdependence, the expectation regarding the penetration level of the other unit will have a large impact on the actual market penetration level of the concerned unit. This phenomenon pertaining to complementary goods is called “indirect network effects” (e.g., Katz and Shapiro, 1994). However, the relationship between the on-board unit and the pedestrian unit differs from that between software and hardware in that each of complementary goods would be owned by different economic group. On the other hand, the relationship is the same as that between vehicle and infrastructure in a hydrogen transportation and nearly identical to the concept of “two-sided network effects” (see, e.g., Rochet and Tirole, 2003).

In this paper, we conduct a theoretical analysis regarding the diffusion policy of the CVPDSSS considering this unique feature. Hereinafter, we will describe the structure and contents of this paper. First, in section 2, we will present the outline of the CVPDSSS. Following this, in section 3, we will conduct theoretical analyses on the equilibrium penetration level of the CVPDSSS and its dynamic feature. In doing so, we will use the model concerning the network externality in Rohlfs (1974). In section 4, we will set concrete functions and illustrate the equilibrium market penetration level of the system and its dynamic feature by diagrams. Finally, in section 5, we will discuss the governmental policy for the promotion of the CVPDSSS.

2 Outline of the Cooperative Vehicle-Pedestrian Driving Safety Support System

Other than the CVPDSSS, the Infrastructure-to-Vehicle (I2V) and the Vehicle-to-Vehicle (V2V) System, whose main aim is to avoid car accidents, also consider
pedestrian-vehicle accidents as one of the most important accident categories to be addressed. In the I2V System, in Japan, the detectors on the roads will pick up pedestrian information and will display the warning on the car navigation equipment via optical beacons on the roads. In the V2V System, the pedestrian information that the sensors on other vehicles have picked up will be sent to the vehicle that may cause an accident. In such ways, it will be possible to avoid accidents between vehicles making a left turn and pedestrians on a pedestrian crossing. However, it should be noted that, in terms of the avoidance of pedestrian-vehicle accidents, both the systems are more suitable for major busy intersections and not very suitable for minor intersections or residential zones. This is because it would be extremely expensive to set up pedestrian detectors and optical beacons at all such areas. In addition, due to the small traffic volume, it will be difficult to obtain sufficient information through the V2V System, which depends on information from other vehicles.

In contrast, the CVPDSSS aims to prevent pedestrian-vehicle accidents at small intersections or residential zones.

There exists a variety of methods concerning the CVPDSSS. In the period between December 2005 and the end of March 2006, NTT DATA Corporation, Nissan Motor Co., Ltd., its communications Inc., TRENDY Corporation, and Tokyu Security Co., Ltd. jointly conducted a demonstration experiment for the CVPDSSS that uses IC tags in the Mitakedai area of Aoba District, Yokohama City. This system aims to ensure the security of children by alerting drivers of the presence of children in the vicinity with the help of a voice recording from an electronic information device in the car that announces the following warning: “Children nearby. Please be careful.” Due to such a warning, the drivers become aware of the presence of children in their blind spots. In this system, both the children and the drivers are required to possess the IC tag. In addition, it is necessary that the
vehicles are equipped with a special electronic information device. The IC signal receivers at intersections and at other places will pick up the IC signals from the children and drivers. The telematics center will then analyze the risk based on the collected information and the concerned driver will be alerted (Fujikura, 2007; Nissan Motor Co., Ltd., 2005). It should be noted that this system does not have a warning function against pedestrians.

Likewise, in Europe, the Watch-Over project started in January 2006. The core of the system is the interaction between an on-board module and a user module. The Watch-Over on board-platform detect pedestrians, cyclists, motorcyclists equipped with the Watch-Over module and warns to the driver providing information only in really dangerous situation. This system also has function to send warnings to vulnerable road users (Meinken et al., 2007; WATCH-OVER Project).

It has been pointed out that one of the most important technical challenges faced with regard to putting the CVPDSSS in practical use is the development of an appropriate man-machine interface through which, for instance, information related to the high probability of an accident is extracted and sent to the driver (e.g., Takeuchi and Maeda, 2007).

3 Equilibrium market penetration level and its dynamic feature

This paper will not address technical challenges of the CVPDSSS such as the man-machine interface problem. Further, we will conduct analyses assuming that all these technical problems are resolved.

As explained above, in the CVPDSSS, both the pedestrians and drivers must possess the special device for this system. The benefit of possessing such a device will depend on the penetration level of the device belonging to the other side. Some technology diffusion models aimed at analyzing the vehicle–infrastructure complementary relationships have been established (e.g., Meyer and Winebrake,
2009; Struben and Sterman, 2008). Meyer and Winebrake (2009) describes a system dynamics model regarding the vehicle–infrastructure relationships exhibited in a hydrogen transportation system. Here, we will conduct theoretical analyses on the equilibrium penetration level of the CVPDSSS as well as on its dynamic feature using the model concerning the network externality in Rohlfs (1974). From a game-theoretic perspective, the basic form of the model we use will be as described below. First, the game players will be pedestrians and drivers. The strategy space for pedestrians will comprise two strategies—(use or do not use) the pedestrian unit and the payoff will be the consumer surplus as a function of each driver’s strategy. In a similar manner, the strategy space for drivers will comprise two strategies—(use or do not use) the on-board unit and the payoff will be the consumer surplus as a function of each pedestrian’s strategy. Neither the pedestrian nor the driver is aware of the payoff function of players other than himself/herself and will select the strategy of the current term under a naive expectation that in the next stage, the other players will select the same strategy that they had chosen in the previous term (We will make a few modifications regarding the formation of such expectations.) This simultaneous game with non-cooperation will be infinitely repeated until the expectation of each player conforms to the self-fulfilling expectation, that is, until the expectation of each player corresponds with the strategy that is chosen in reality. The final equilibrium market penetration level will be equal to the stationary equilibrium penetration level of this super game, so called, the Nash Equilibrium.

3. 1 Major assumptions

In this section, we will first explain the major assumptions of the analysis model used in this paper.
3.1.1 Each pedestrian’s (driver’s) decision making

In this paper, for the purpose of simplification, we will conduct the analysis under the assumption that the pedestrian unit and the on-board unit are different goods. At the stage for practical use, there is a possibility that the pedestrian unit will also function (or partly function) as the on-board unit. However, we anticipate that the majority of people possessing the pedestrian unit would be school children or elderly persons and in practice, one pedestrian unit will be rarely used as an on-board unit (or a part of such a device) as well. We thus consider that this assumption will not raise questions regarding the validity of the result of the analyses.

Second, in this paper, we assume that all pedestrians and drivers are homogeneous from the standpoint of drivers and pedestrians, respectively. In the case of pedestrians, the benefit is, as we explained above, the function of each driver’s strategy (in other words, each driver’s use or non-use of the on-board unit). However, under this assumption, the benefit of each pedestrian will be expressed simply as a function of the penetration level of the on-board unit and not as a function of each driver’s strategy. In a similar manner, the benefit of each driver will be expressed simply as a function of the penetration level of the pedestrian unit.

Based on the above assumptions, whether or not one pedestrian \( i \) (in many cases, the guardian of pedestrian \( i \)) uses the pedestrian unit shall be decided by the following conditional equation:

\[
\begin{align*}
\alpha_{f,i}(p^e_B) = p^e_B u_{f,i} < c_f & \rightarrow \text{not use} \\
\alpha_{f,i}(p^e_B) = p^e_B u_{f,i} \geq c_f & \rightarrow \text{use}
\end{align*}
\]

where

\( u_{f,i} \) is the benefit that pedestrian \( i \) can enjoy by using the unit for a certain period (hereinafter “one term”) when the penetration level of the on-board unit is 1,
\( p_v^e \) is the expectation of pedestrians regarding the penetration level of the on-board unit \((0 \leq p_v^e \leq 1)\). We assume that all pedestrians have the same expectation level, \( u_{f,i}(p_v^e) \) is the benefit that pedestrian \( i \) expects to enjoy by using the pedestrian unit for one term when the expected penetration level of the on-board unit is \( p_v^e \). We assume that the expected benefit will change in proportion to the expected penetration level of the on-board unit, and \( c_f \) is the rental cost of the pedestrian unit. We assume that the production function of the pedestrian unit displays constant returns to scale and that the rental cost is equal to the amount dividing the price of the pedestrian unit by the period of endurance. Based on this assumption, the rental cost will be constant regardless of the penetration level. Accordingly, in this paper, we will mention cases in which the pedestrian unit or the on-board unit is purchased.

In this paper, we use subscript “\( f \)” for pedestrians or pedestrian units and subscript “\( v \)” for drivers or on-board units. We also clarify the equations for pedestrians or pedestrian units by the letter “\( f \)” after equation numbers and the equations for drivers or on-board units by letter “\( v \)”.

Based on these assumptions, Eq. (1f) shows that pedestrian \( i \) will use the pedestrian unit when the expected benefit for using the unit is higher than or equal to \( c_f \) and will not use it when the expected benefit is smaller than \( c_f \).

On the other hand, with regard to each driver’s use of an on-board unit, under the above assumptions, it will be possible to express it in a mathematically symmetrical manner to the pedestrian. Whether or not driver \( j \) uses the on-board unit will depend on the following conditional equation:

\[
\begin{cases}
  u_{v,j}(p_f^e) = p_f^e u_{v,j} < c_v \rightarrow \text{not use} \\
  u_{v,j}(p_f^e) = p_f^e u_{v,j} \geq c_v \rightarrow \text{use}
\end{cases}
\]

where
\(u_{v,j}\) is the benefit that driver \(j\) can enjoy by using the on-board unit for one term when the penetration level of the pedestrian unit is 1, 

\(p^e_f\) is the expectation of drivers regarding the penetration level of the pedestrian unit \((0 \leq p^e_f \leq 1)\),

\(u_{v,j}(p^e_f)\) is the benefit that driver \(j\) expects to enjoy by using the on-board unit for one term when the expected penetration level of the pedestrian unit is \(p^e_f\), and

\(c_v\) is the rental cost of the on-board unit.

As explained above, the conditional equations for the pedestrian’s use of the pedestrian unit and the driver’s use of the on-board unit have a symmetrical feature. We will explain mainly about the pedestrian unit below.

### 3.1.2 Expectation formation related to the penetration level

We assume that \(p^e_v\) (the expectation of pedestrians related to the penetration level of the on-board unit) can be expressed as a function of the actual penetration level of the on-board unit in the previous term as follows:

\[
p^e_v = E_v(p_{v,-1}). \tag{2v}
\]

We assume that \(\frac{d}{dp_{v,-1}} E_v \geq 0\).

In a similar manner, we assume that \(p^e_f\) can be expressed as a function of the actual penetration level of the pedestrian unit in the previous term as follows:

\[
p^e_f = E_f(p_{f,-1}). \tag{2f}
\]

We assume that \(\frac{d}{dp_{f,-1}} E_f \geq 0\).

### 3.2 Demand function

The benefits of using the pedestrian unit will be different among individuals.
If we consecutively number pedestrians from 1, 2⋯⋯n, beginning with the pedestrian who enjoys the largest benefit under a certain penetration level of the on-board unit and plot them by setting the pedestrian numbers on the horizontal axis and the benefit to the pedestrians on the vertical axis, we can draw the demand curve for the pedestrian units. If we change the horizontal axis from the pedestrian numbers to the penetration level, \( p_f \), the inverse demand function of a pedestrian unit (the marginal benefit function) will be as follows:

\[
u_f = D_f(p_v^e, p_f) = a_f p_v^e U_f(p_f),
\]

where \( a_f \) shows the benefit level for the pedestrian who enjoys the largest benefit by using the pedestrian unit when the penetration level of the on-board unit is 1. We can obtain \( U_f(p_f) \) by dividing each pedestrian’s benefit when the penetration level of the on-board unit is 1 by \( a_f (=u_{f,i}/a_f) \) and arranging them in a descending order and by changing the horizontal axis from the pedestrian numbers to the penetration level of the pedestrian unit, \( p_f \). Under this definition, \( U_f(0) = 1 \) and \( \frac{d}{dp_f} U_f(p_f) \leq 0 \) will be established.

In addition, the inverse demand function of the on-board unit will be, symmetrical to the pedestrian unit, as follows:

\[
u_v = D_v(p_v^e, p_v) = a_v p_f^e U_v(p_v),
\]

where \( p_v \) is the penetration level of the on-board unit, and \( a_v \) shows the benefit level for the driver who enjoys the largest benefit by using the on-board unit when the penetration level of the pedestrian unit is 1. Under this definition, \( U_v(0) = 1 \) and \( \frac{d}{dp_v} U_v(p_v) \leq 0 \) will be established.

3.3 Penetration level in each term and the minimum penetration level

3.3.1 The penetration level in each term and the reaction curve
Based on Eqs. (3f) and (2v), the penetration level of the pedestrian unit in each term $p_{f,t}$ can be expressed as the solution of the following difference equation:

$$F_f(p_{v,t},p_{f,t},c_f) |_{p_{v,t}=E_v(p_{v,t-1})} = \frac{1}{a_f} (D_f(p_{v,t},p_{f,t}) |_{p_{v,t}=E_v(p_{v,t-1})} - c_f)$$

$$= E_v(p_{v,t-1}) U_f(p_{f,t}) - r_f = 0, \quad (4f)$$

where $r_f$ is the ratio between the benefit level for the pedestrian who enjoys the largest benefit by using the pedestrian unit when the penetration level of the on-board unit is 1 and the rental cost of the pedestrian unit ($c_f/a_f$), and usually, $r_f < 1$ will be established. Here, we assume that Eq. (4f) implicitly defines the following function:

$$p_{f,t} = G_f(p_{v,t-1}, r_f). \quad (5f)$$

Thus far, we have described the model of this paper as a game in which all the pedestrians and drivers are players. However, in reality, this game is nothing but the game in which the representatives of the pedestrians and the drivers are players, and the strategy space and the payoff for the representative of the pedestrians are the rate of utilization of the pedestrian unit (= the penetration level) and the sum of consumer surplus of pedestrians as a whole, respectively. Further, the strategy space and the payoff for the representative of the drivers are the rate of utilization of the on-board unit and the sum of consumer surplus of drivers as a whole, respectively. Eq. (5f) denotes the reaction curve for the representative of pedestrians in a game formulated as above. Here, the slope of the reaction curve of pedestrians $\frac{\partial p_{f,t}}{\partial p_{v,t-1}}$ will be expressed as follows:

$$\frac{\partial p_{f,t}}{\partial p_{v,t-1}} = - \frac{\frac{\partial F_f}{\partial p_{v,t-1}}}{\frac{\partial F_f}{\partial p_{f,t}}} = - \frac{U_f(p_{f,t})}{E_v(p_{v,t-1})} \frac{dE_v}{dp_{v,t-1}} \frac{dU_f}{dp_{f,t}}$$
\[ - \frac{p_{f,t}}{p_{v,t-1}} \frac{\eta_{E_v}(p_{v,t-1})}{\eta_{U_f}(p_{f,t})}, \]  

(56)

where \( \eta_{U_f} \) and \( \eta_{E_v} \) are the point elasticities for \( U_f \) and \( E_v \), respectively. As \( \eta_{U_f} \leq 0 \) will be established under the definition and \( \eta_{E_v} \geq 0 \) will be established under the assumption, the slope of the reaction curve \( \frac{\partial p_{f,t}}{\partial p_{v,t-1}} \) is nonnegative.

In addition, the equations for the on-board unit, which correspond to Eqs. (4f), (5f), and (6f) of the pedestrian unit will be as follows:

\[ F_v(p_{f,t}, p_{v,t}, c_v) |_{p_{f,t}=E_v(p_{f,t-1})} = \frac{1}{a_v}(D_v(p_{f,t}, p_{v,t}) |_{p_{f,t}=E_v(p_{f,t-1})} - c_v) \]

\[ = E_f(p_{f,t-1}) U_v(p_{v,t}) - r_v = 0 \quad \text{where} \quad r_v = \frac{c_v}{a_v}, \]  

(4v)

\[ p_{v,t} = G_v(p_{f,t-1}, r_v), \]  

(5v)

\[ \frac{\partial p_{v,t}}{\partial p_{f,t-1}} = - \frac{p_{v,t}}{p_{f,t-1}} \frac{\eta_{E_v}(p_{f,t-1})}{\eta_{U_f}(p_{v,t})}. \]  

(6v)

Similar to the pedestrian unit, the slope of the reaction curve for drivers, \( \frac{\partial p_{v,t}}{\partial p_{f,t-1}} \), is nonnegative.

### 3.3.2 The minimum penetration level

Under Eq. (4f), we assume that the penetration level of the on-board unit, \( p_{v}^{\min} \) \( (0 \leq p_{v}^{\min} \leq 1) \), which satisfies the following equation exists:

\[ F_f(p_{v}^{\min}, 0, c_f) |_{p_{v}^{\min}=E_v(p_{v}^{\min})} = 0, \quad \text{that is} \quad E_v(p_{v}^{\min}) = r_f. \]  

(7f)

\( p_{v}^{\min} \) shows the minimum penetration level of the on-board unit as there exist pedestrians who rent the pedestrian unit. If the penetration level of the on-board unit is lower than this value, no pedestrian will rent the pedestrian unit.

In a similar manner, from Eq. (4v), we can calculate the minimum penetration level of the pedestrian unit by using the following equation:
The penetration level of the pedestrian unit

<table>
<thead>
<tr>
<th>Initial</th>
<th>1st term</th>
<th>2nd term</th>
<th>3rd term</th>
<th>4th term</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{f,0}$</td>
<td>$p_{f,1}$</td>
<td>$p_{f,2}$</td>
<td>$p_{f,3}$</td>
<td>$p_{f,4}$</td>
<td>...</td>
</tr>
</tbody>
</table>

The penetration level of the on-board unit

<table>
<thead>
<tr>
<th>Initial</th>
<th>1st term</th>
<th>2nd term</th>
<th>3rd term</th>
<th>4th term</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{v,0}$</td>
<td>$p_{v,1}$</td>
<td>$p_{v,2}$</td>
<td>$p_{v,3}$</td>
<td>$p_{v,4}$</td>
<td>...</td>
</tr>
</tbody>
</table>

Fig. 1  Two sequences for the penetration level

$$F_v\left(p_f^{min},0,c_v\right)|_{p_f^{min}=E_f(p_f^{min})} = 0$$, that is, $$E_f\left(p_f^{min}\right) = r_v$$.

3.4 Dynamic alteration of the penetration level

There is no guarantee that $p_{v,t}$ determined by the reaction curve for drivers (Eq. (5v)) corresponds to $p_{v,t}^o$. If there is discordance between them, the pedestrians will revise their expectation and the demand curve for the pedestrian (3f) will shift.

If we substitute Eq. (5v) for Eq. (4f) and Eq. (5f) for Eq. (4v), we can obtain the following difference equations:

$$H_f(p_{f,t},p_{f,t-2},r_f,r_v) = E_v(p_{v,t-1})U_f(p_{f,t})|_{p_{v,t-1}=G_v(p_{v,t-2},r_v)} - r_f = 0$$

(8f)

$$H_v(p_{v,t},p_{v,t-2},r_v,r_f) = E_f(p_{f,t-1})U_v(p_{v,t})|_{p_{f,t-1}=G_f(p_{f,t-2},r_f)} - r_v = 0$$

(8v)

Based on Eqs. (8f) and (8v), we can understand that for both the pedestrian units and the on-board units, the penetration level of this term will be influenced by their penetration levels in the term before the previous term. However, they will not be influenced by the penetration level in the previous term. If we present a graphic representation, as shown in Fig. 1, the penetration level of each term will be the combination of two sequences, heavy-line sequence and dot-line sequence.

Based on Eq. (8f), we can draw a phase diagram that has the penetration level of
the pedestrian unit in the term before the previous term as the horizontal axis and
the penetration level of the pedestrian unit in the current term as the vertical axis.

Based on Eqs. (8f) and (6v), the slope of the phase line, \( \alpha_f \), will be described as follows:

\[
\alpha_f(p_{f,t}, p_{f,t-2}, p_{v,t-1}, r_v) = \frac{\partial p_{f,t}}{\partial p_{f,t-2}} = -\frac{\partial H_f}{\partial p_{f,t}} = \frac{U_f(p_{f,t})}{E_v(p_{v,t-1})} \frac{\partial p_{v,t-1}}{\partial p_{f,t}} \eta_{E_v}(p_{v,t-1}) \eta_{U_v}(p_{f,t}) \frac{\partial p_{v,t-1}}{\partial p_{f,t-2}}
\]

\[
\alpha_v(p_{v,t}, p_{v,t-2}, p_{f,t-1}, r_f) = \frac{\partial p_{v,t}}{\partial p_{v,t-2}}
\]

\[
\frac{p_{v,t}}{p_{v,t-2}} \eta_{E_v}(p_{v,t-1}) \eta_{U_v}(p_{f,t}) \eta_{U_v}(p_{v,t-1}) \geq 0.
\]

As \( \eta_{U_v} \leq 0 \) and \( \eta_{U_v} \leq 0 \) will be established under the definition and \( \eta_{E_v} \geq 0 \) and \( \eta_{E_v} \geq 0 \) will be established under the assumption, \( \alpha_f \) is nonnegative.

In a similar manner, the slope of the phase line for the on-board unit, \( \alpha_v \), will be as follows:

\[\]

3.5 Stationary equilibrium penetration level

Here, we assume that Eqs. (8f) and (8v) have stationary equilibrium penetration levels \( \bar{p}_f \) and \( \bar{p}_v \), \( \bar{p}_f \), \( \bar{p}_v \), satisfying the following Eqs. (10f) and (10v):

\[H_f(p_f, \bar{p}_f, r_f, r_v) = E_v(\bar{p}_v) U_f(\bar{p}_f) |_{|_{E_v=G_v(\bar{p}_v, r_v)}} - r_f = 0, \quad (10f)\]

\[H_v(p_v, p_v, r_v, r_f) = E_f(p_f) U_v(p_v) |_{|_{E_f=G_f(\bar{p}_v, r_v)}} - r_v = 0. \quad (10v)\]
People will continue to modify their expectations until these stationary equilibrium penetration levels is realized, causing the demand curves (Eqs. (3f) and (3v)) to continue to shift. However, people will not make further modifications after the penetration levels reach these stationary equilibrium penetration levels.

From a game-theoretic perspective, this equilibrium penetration level implies nothing other than the intersecting point of the reaction curve for pedestrians (Eq. (5f)) and the reaction curve for drivers (Eq. (5v)), that is, the Nash Equilibrium.

Here, if we calculate the ratio between the slope of the reaction curve for drivers (Eq. (6v)) and the inverse of the slope of the reaction curve for pedestrians (Eq. (6f)) at the stationary equilibrium penetration level, the following equation will be established:

\[
\frac{\frac{\partial p_{v,t}}{\partial p_{f,t-1}}}{\frac{1}{\frac{\partial p_{f,t}}{\partial p_{v,t-1}}}} \bigg|_{p_{f,t}=p_{f,t-1}, p_{v,t}=p_{v,t-1}} = \frac{\eta_{E_v}(\bar{p}_v)}{\eta_{U_f}(\bar{p}_f)} \frac{\eta_{E_f}(\bar{p}_f)}{\eta_{U_v}(\bar{p}_v)}
\]

\[= \alpha_f(\bar{p}_f, \bar{p}_v, r_f) = \alpha_v(\bar{p}_v, \bar{p}_f, r_v, r_f). \quad (11)\]

Therefore, at the stationary equilibrium penetration level, if the slope of the reaction curve for drivers (Eq. (6v)) is smaller than the inverse of the slope of the reaction curve for pedestrians (Eq. (6f)), \(\alpha_f(\bar{p}_f, \bar{p}_v, r_v) = \alpha_v(\bar{p}_v, \bar{p}_f, r_f) < 1\) will be established and the stationary equilibrium penetration level will be dynamically stable. On the other hand, if the slope of the reaction curve for drivers is not smaller than the inverse of the slope of the reaction curve for pedestrians, \(\alpha_f(\bar{p}_f, \bar{p}_v, r_v) = \alpha_v(\bar{p}_v, \bar{p}_f, r_f) \geq 1\) will be established and the stationary equilibrium penetration level will be dynamically unstable.

4 Graphic representation

In this section, we will graphically show the stationary equilibrium penetration level and its dynamic feature by presenting concrete functions in the expectation...
formation function $E_f$, $E_v$ as well as in demand functions $U_f$, $U_v$.

4.1 Formulation of the expectation formation and the demand functions

In order to conduct the analysis, we assume that, both for the on-board and the pedestrian units, the expected penetration levels in the current term ($p_{v,t}^e$ and $p_{f,t}^e$) are equal to the realized market penetration levels in the previous term ($p_{v,t-1}$ and $p_{f,t-1}$). Likewise, the benefit for pedestrians using the pedestrian unit and the benefit for drivers using the on-board unit will be identically distributed in intervals of [$\alpha_f p_v^e (1-b_f)$, $\alpha_f p_v^e$] and [$\alpha_v p_f^e (1-b_v)$, $\alpha_v p_f^e$], respectively (see Eqs. (3f') and (3v')). Under these two assumptions, equations related to the pedestrian unit—Eqs. (2v), (3f), (4f), (5f), (7f), (8f), and (10f)—can be reformulated as follows:

\[
p_{v,t}^e = p_{v,t-1}, \quad (2v')
\]

\[
u_f = a_f p_v^e (1-b_f p_f), \quad (3f')
\]

\[
p_{v,t-1}(1-b_f p_{f,t})-r_f = 0, \quad (4f')
\]

\[
p_{f,t} = \frac{1}{b_f} \left(1-r_f \frac{1}{p_{v,t-1}}\right), \quad (5f')
\]

\[
p_{v}^{\text{min}} = r_f, \quad (7f')
\]

\[
\frac{1}{b_v} \left(1-r_v \frac{1}{p_{f,t-2}}\right)(1-b_f p_{f,t})-r_f = 0, \quad (8f')
\]

\[
\frac{1}{b_v} \left(1-r_v \frac{1}{p_f}\right)(1-b_f p_f)-r_f = 0. \quad (10f')
\]

Further, as for the point elasticity for $U_f$, the following equation will be established:

\[
\eta_{U_f} = \frac{b_f p_f}{b_f p_f - 1}. \quad (12f)
\]

In a similar manner, with regard to the on-board unit, under the same assumptions,
Eqs. (2f), (3v), (4v), (5v), (7v), (8v), and (10v) can be reformulated as follows:

\[ p_{f,t}^e = p_{f,t-1}, \quad (2f') \]
\[ u_v = a_v p_{f,t}^e (1 - b_v p_v), \quad (3v') \]
\[ p_{f,t-1} (1 - b_v p_v,t) - r_v = 0, \quad (4v') \]
\[ p_{v,t} = \frac{1}{b_v} \left(1 - r_v \frac{1}{p_{f,t-1}}\right), \quad (5v') \]
\[ p_{f}^{\min} = r_v, \quad (7v') \]
\[ \frac{1}{b_f} \left(1 - r_f \frac{1}{p_{v,t-2}}\right) (1 - b_v p_{v,t}) - r_v = 0, \quad (8v') \]
\[ \frac{1}{b_f} \left(1 - r_f \frac{1}{p_v}\right) (1 - b_v p_v) - r_v = 0. \quad (10v') \]

Further, as for the point elasticity for \( U_v \), the following equation will be established:

\[ \eta_{U_v} = \frac{b_v p_v}{b_v p_v - 1}. \quad (12v) \]

4.2 Base case

Here, we will first conduct the analysis by setting \( b_f = b_v = 1, r_f = 0.15 \) and \( r_v = 0.25 \). Hereinafter, we will refer to this case as the “base case.”

4.2.1 The reaction curve

In Fig. 2, ReactionCurve\(_f\) and ReactionCurve\(_v\) give the reaction curve for pedestrians (Eq. (5f')) and that for drivers (Eq. (5v')), respectively. ReactionCurve\(_f\) is a curve that has \( p_{v,t-1} \) as the explaining variable and \( p_{f,t} \) as the explained variable. In this case, we have reversed the graph and set \( p_{f,t} \) as the horizontal axis and \( p_{v,t-1} \) as the vertical axis. ReactionCurve\(_f\) intersects with the vertical axis at \( p_v^{\min} \)
which shows the minimum penetration level of the on-board unit and from Eq. (7f'),
this is equal to $r_f$. As long as $p_{v,t-1}$ is smaller than this minimum penetration level,
$p_{f,t}$ will be zero.

On the other hand, $ReactionCurve_v$ is a curve that has $p_{f,t-1}$ as the explaining
variable and $p_{v,t}$ as the explained variable. In this case, we have drawn the graph
by setting $p_{f,t-1}$ as the horizontal axis and $p_{v,t}$ as the vertical axis. $ReactionCurve_v$
intersects with the horizontal axis at $p_{f,\text{min}}$ which shows the minimum penetration
level of the pedestrian unit and from Eq. (7v'), this is equal to $r_v$. As long as $p_{f,t-1}$ is

---

Fig. 2  Reaction curve (Base case)

Note: Fig. drawn under $b_f=b_v=1$, $r_f=0.15$, $r_v=0.25$, and $r_v'=0.20$. 
smaller than this minimum penetration level, $p_{v,t}$ will be zero.

In Fig. 2, ReactionCurve$_f$ and ReactionCurve$_v$ intersect at three points, which are $O$, $E_1$, and $E_2$. Therefore, there are three Nash equilibria. However, from the point of view of consumer surplus, as $O < E_1 < E_2$ will be established, $E_2$ will be the desirable equilibrium point.

Let us now assume that $r_v$ has decreased to $r_v'$ which is equal to 0.20. As $r_v$ decreases, from Eq. (7v'), the minimum penetration level of the pedestrian unit will decrease from $p_{f}^{\text{min}}$ to $p_{f}^{\text{min}'_v}$. Then the reaction curve for drivers will shift upward to ReactionCurve$_v'$. As a result, $E_1$, $E_2$ will shift to $E_1'$, $E_2'$, respectively.

4.2.2 Dynamic feature

$E_1$ corresponds to the penetration level, which is referred to as “critical mass” in the theory of the network externality. If the penetration level of the current term increases to slightly higher than this level, the market penetration level will automatically converge to $E_2$. Let us look at this feature in the phase diagram.

In the upper panel in Fig. 3, ReactionCurve$_v$ and PhaseLine$_f$ give the reaction curve for drivers (Eq. (5v')) and the phase line for pedestrians (Eq. (8f')), respectively. The straight thin line is the forty-five degree line. $A$ and $B$ in this figure correspond to $E_1$ and $E_2$ in Fig. 2, respectively. On the other hand, ReactionCurve$_f$ and PhaseLine$_v$ in the lower panel in Fig. 3 show the reaction curve for pedestrians (Eq. (5f')) and the phase line for drivers (Eq. (8v')), respectively. $C$ and $D$ in this figure correspond respectively $E_1$ and $E_2$ in Fig. 2, respectively.

When we look at the upper panel in Fig. 3, $p_{f}^*$ shows the penetration level of the pedestrian unit of the term before the previous term, which makes $p_{f,t}$ zero and $p_{f}^{\text{min}} < p_{f}^*$ will be established. At $E_1$ in Fig. 2, the slope of ReactionCurve$_v$ is greater than the inverse of the slope of ReactionCurve$_f$. Therefore, at $A$ in Fig. 3, the slope of PhaseLine$_f$, $\alpha_f$, is greater than 1 from Eq. (11), which means $E_1$ is
Fig. 3  Phase diagram (Base case)

Note: Fig. drawn under $b_f = b_v = 1$, $r_f = 0.15$, and $r_v = 0.25$. 
a dynamically unstable equilibrium point. On the other hand, at $E_2$ in Fig. 2, the slope of $ReactionCurve_v$ is smaller than the inverse of the slope of $ReactionCurve_v$. Therefore, at $B$ in Fig. 3, the slope of $PhaseLine_f$, $\alpha_f$, is smaller than 1 from Eq. (11), which means $E_2$ is a dynamically stable equilibrium point. While the proof is beyond the scope of this paper, $PhaseLine_f$ is concave. Therefore, if $p_{f,t-2}$ exists in the area of $p_{f,t-2} < \bar{p}_f^1$, the market penetration level will converge to zero. On the other hand, if $p_{f,t-2}$ exists in $p_{f,t-2} < \bar{p}_f^1$, the market penetration level will converge to $\bar{p}_f^2$. Therefore, if the default values of the pedestrian unit and the on-board unit $(p_{f,0}, p_{v,0})$ are slightly higher than the critical mass, both the sequences in Fig. 1 will converge to $(\bar{p}_f^2, \bar{p}_v^2)$. If we refer to the example of the pedestrian unit, the sequences will be expressed as follows:

\begin{align*}
\text{Heavy line sequence in Fig. 1} & \quad p_{f,0} < p_{f,3} < \ldots \ldots \rightarrow \bar{p}_f^2, \\
\text{Dotted line sequence in Fig. 1} & \quad p_{f,1} < p_{f,3} < \ldots \ldots \rightarrow \bar{p}_f^2.
\end{align*}

In this case, $p_{f,0} < p_{f,1} < p_{f,2} < p_{f,3}$ will not be established and the penetration level will oscillate. However, the oscillation is a unique phenomenon that occurs in a case wherein pedestrians rent the unit and decide whether or not to rent the pedestrian unit at the beginning of every term. When pedestrians purchase the unit, the penetration level shows downward rigidity at the maximum penetration level realized and oscillation does not occur.

4.3 Inelastic case

Depending on the value of $\eta_{U_f}$ (the point elasticity of $U_f$) or $\eta_{U_v}$ (the point elasticity of $U_v$), the combination of market penetration levels other than $(\bar{p}_f, \bar{p}_v)$ which satisfies Eqs. (10f) and (10v) can be the desirable equilibrium point. In order to show that, we conduct the analysis again by setting $b_f = 1$, $b_v = 0.6$, $r_f = 0.15$, and $r_v = 0.25$. In this case, $U_v$ is inelastic as compared to the base case. Hereinafter, we will refer to this case as the “inelastic case.”
4.3.1 The reaction curve

In Fig. 4, Reaction\textit{Curve}_f and Reaction\textit{Curve}_v give the reaction curve for pedestrians (Eq. (5f')) and that for drivers (Eq. (5v')) in the inelastic case, respectively.

In this case, there will be three Nash equilibria—O, E_1 and E_3. In Fig. 4, E_2 is indicated as another stationary equilibrium point. However, this penetration level will never be realized as \( \bar{p}_v^2 \) which corresponds to \( \bar{p}_f^2 \) exceeds 1.

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**Note:** Fig. drawn under \( b_f = 1, b_v = 0.6, r_f = 0.15, \) and \( r_v = 0.25. \)
4.3.2 The dynamic feature

At $E_1$ in Fig. 4, the slope of $Reaction Curve_v$ is greater than the inverse of the slope of $Reaction Curve_f$. Therefore, based on Eq. (11), the slope of the phase diagram for the pedestrian unit, $\alpha_f$, and that for the on-board unit, $\alpha_v$, are greater than 1. $E_1$ thus show a dynamically unstable critical mass.

In Fig. 4, $p_f^{**}$ is the penetration level of the pedestrian unit in the previous term, which makes $p_{v,t}$ 1. The penetration level of the pedestrian unit in the next term which corresponds to the 100 per cent penetration of the on-board unit in the current term is $p_f^{\text{max}}$. Therefore, if $p_{f,t-2}$ exists in $\overline{p_f^1} < p_{f,t-2} < p_f^{\text{max}}$, the dynamic alteration of the market penetration level will terminate when it reaches $E_3$.

5 Diffusion policies

Finally, let us consider the governmental policies for the promotion of the CVPDSSS under the assumption that the critical mass exists.

As explained in the abovementioned analyses, once both the penetration level of the pedestrian unit and that of the on-board unit exceed the critical mass, both the market penetration levels will attain desirable stable equilibrium points through the lasting mechanism. Therefore, as a first policy, the government may consider freely distributing units at the level that slightly exceeds the critical mass. However, the government does not need to freely distribute both the pedestrian and the on-board units. Let us consider the case in which we distribute pedestrian units at the level $p_{f,0}^g$ without charge when the penetration level of the on-board unit is zero. In this case, from a theoretical perspective, the two sequences in Fig. 1 will be expressed as follows:

Heavy line sequence in Fig. 1: $p_{f,0}(=p_{f,0}^g)<p_{f,2}<p_{f,4}$ \ldots \ldots \rightarrow $p_f^{opt}$,

Dotted line sequence in Fig. 1: $p_{f,1}(=p_{v,0})=p_{f,3}=p_{f,5}$ \ldots \ldots = 0,

Where $p_f^{opt}$ is the desirable stable equilibrium penetration level of the pedestrian
unit which corresponds to \( p^g_f \) in Fig. 2 and \( p^\text{max}_f \) in Fig. 4. However, as pedestrian units at the level \( p^g_{f,0} \) are freely distributed, the penetration level of the pedestrian unit shows the downward rigidity at \( p^g_{f,0} \), and the two sequences in Fig. 1 will be expressed as follows:

- **Heavy line sequence in Fig. 1:** \( p_{f,0}(=p^g_{f,0})<p_{f,2}<p_{f,A} \cdots \cdots \cdots \rightarrow p^{\text{opt}}_f \),
- **Dotted line sequence in Fig. 1:** \( p_{f,1}=p_{f,3}=p_{f,5} \cdots \cdots \cdots =p^g_{f,0} \).

In addition, when we consider the case in which pedestrian units are purchased as well, as the penetration level shows the downward rigidity at the maximum penetration level once realized, the two sequences in Fig. 1 will be expressed as follows:

- **Heavy line sequence in Fig. 1:** \( p_{f,0}(=p^g_{f,0})<p_{f,2}<p_{f,A} \cdots \cdots \cdots \rightarrow p^{\text{opt}}_f \),
- **Dotted line sequence in Fig. 1:** \( p_{f,1}=p_{f,0}, p_{f,3}=p_{f,2}, p_{f,5}=p_{f,A} \rightarrow p^{\text{opt}}_f \).

As a matter of course, as the pedestrian unit converges to \( p^{\text{opt}}_f \), the on-board unit will also converge to the desirable penetration level, \( p^{\text{opt}}_f \).

As the second policy, we will consider the case in which the government sets the target penetration level for the pedestrian unit and wherein the formation of the expectation of drivers regarding the penetration level of the pedestrian unit is influenced by this target setting. We will then modify Eq. \((2f')\) as follows:

\[
p^E_{f,t} = E_f(p_{f,t-1}) = p_{f,t-1} + \delta_f(p^g_f-p_{f,t-1}) = (1-\delta_f)p_{f,t-1} + \delta_f p^g_f, \tag{2f''}
\]

where \( p^g_f \) shows the targeted penetration level of the pedestrian unit by the government and \( \delta_f (0<\delta_f \leq 1) \) shows the speed of adjustment of \( p^E_{f,t} \) to \( p^g_f \). In this case, with regard to the elasticity of \( E_f \),

\[
\eta_{E_f}(p_{f,t-1}) = \frac{(1-\delta_f)p_{f,t-1}}{(1-\delta_f)p_{f,t-1} + \delta_f p^g_f} < 1 \tag{13}
\]

will be established. In the base case in which the expected penetration level of
the current term is equal to the actual penetration level of the previous terms, \( \eta_{E_f}(p_{f,t-1}) = 1 \) is established. Therefore, the value of Eq. (13) will be always smaller than the base case. On the other hand, as we can understand from Eq. (9f) and (9v), when \( \eta_{E_f} \) becomes smaller, the slopes of the phase line will be smaller for both the pedestrian and the on-board units.

Fig. 5 shows the phase diagram for the pedestrian unit (upper panel) and the phase diagram for the on-board unit (lower panel) when we set \( b_f = b_v = 1 \), \( r_f = 0.15 \), \( r_v = 0.25 \), and \( \delta_f = 0.2 \) and set \( \bar{p}_{f}^0 \) equal to \( \bar{p}_{f}^2 \) in Fig. 3 (the desirable penetration level of the base case). The values of \( b_f, b_v, r_f, \) and \( r_v \) are equal to those of base case. In the case of both the upper panel and the lower panel, the heavy line curve is the phase line showing the relationship between the penetration level in the term before the previous term and the penetration level of the current term and the thin line is the reaction curve. When we look at the phase line, the slope of the phase line is somewhat gentler as compared to that of the base case (Fig. 3). This implies that the timing of convergence to the desirable penetration levels will accelerate as compared to the case in which the target is not set by the government. Therefore, the amount of deadweight loss that occurs in the course of realizing the stationary equilibrium penetration level can also be decreased.

It should be also noted that in the case of both the pedestrian and the on-board units, the critical mass will be also smaller than that in the base case; that is to say \( \bar{p}_{f}^1 \) in Fig. 5 < \( \bar{p}_{f}^2 \) in Fig. 3 and \( \bar{p}_{v}^1 \) in Fig. 5 < \( \bar{p}_{v}^2 \) in Fig. 3 will be established. This implies that the government can reduce the scale of free distributions required to achieve the desirable penetration level, that is, it can reduce the financial cost.

Meanwhile, when we set the target penetration level for the pedestrian unit, we have to consider whether the government should distribute the pedestrian unit or the on-board unit without charge. It depends on the levels of pedestrian unit and on-board unit at critical mass, the numbers of pedestrians and drivers, as well as
Fig. 5  Phase diagram (The case in which the government has set the targeted penetration level of the pedestrian unit)

Note: Fig. drawn under $b_f = b_v = 1$, $r_f = 0.15$, $r_v = 0.25$, and $\delta_f = 0.2$. $p_f^*\text{ min}$ is set equal to $\bar{p}_f^*$ in Fig 3 (the desirable penetration level of the base case).
the prices of the pedestrian and on-board units. However, from the point of view of increasing $\delta_f$, which shows the speed of adjustment of $p_f^e$ to $p_f^g$, the government should distribute pedestrian units for free.

6 Concluding remarks

In this paper, we have conducted theoretical analyses on the market equilibrium penetration level for the CVPDSSS and its dynamic feature. In doing so, we have used the model concerning the network externality in Rohlfs (1974). Based on the result of these analyses, we have discussed the governmental policy to promote the CVPDSSS and found the following. (1) If there exists a critical mass for pedestrian units and on-board units, the government should be able to achieve the desirable penetration level of the CVPDSSS by distributing free pedestrian units (or on-board units) in a number that slightly exceeds its critical mass. (2) We will be able to reduce the financial cost for the distribution of free units by reducing the critical mass through a public announcement of the target penetration level.

With regard to the choice between the pedestrian and the on-board units in order to set the target penetration level, the quantity of free distribution, the target for the free distribution, and so on, it will be necessary to modify the assumption of the homogeneity of pedestrians and drivers and we need more detailed analyses by setting concrete values in parameters. We would like to conduct such analyses in a different paper. However, in our view, considering the social acceptability of the policy, it will be realistic to set the target penetration level for the pedestrian unit and distribute pedestrian units to kindergarten and elementary school children or elderly persons.
【References】


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Abstract

Hiroaki MIYOSHI and Masayoshi TANISHITA, *Diffusion Policy for the Cooperative Vehicle–Pedestrian Driving Safety Support System*

The cooperative vehicle–pedestrian driving safety support system is a system connecting pedestrians and vehicles through signal communications in order to alert drivers of the presence of pedestrians in their blind spots. In this paper, we conduct a theoretical examination of the governmental policy necessary for the promotion of this system. The main conclusions of this paper are as follows. The government should be able to diffuse the cooperative vehicle–pedestrian driving safety support system into a society by distributing free pedestrian units (or on-board units) in a number that slightly exceeds the penetration critical mass level. Likewise, we are able to reduce the financial cost for the distribution of free units by reducing the critical mass through a public announcement regarding the target penetration level.