# A Model of Macrodynamics with an Expectations Hypothesis*

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## I Introduction

As is well known, the word 'uncertainty' has been given two different meanings in economics. It has been used, on the one hand, to mean the state when people do not know future events *perfectly* but their probabilistic distribution, what F.K. Knight called 'measurable uncertainty', or simply 'risk'. On the other hand, it has been considered to be the state when people have little or none of the information needed to perform any probabilistic calculations, what has been called 'true uncertainty' or possibly 'Keynes/Knightian uncertainty'.[1]

It is, needless to say, of crucial importance whether or not people can know the probability distribution of future events in advance. Under the principles of Keynes/Knightian uncertainty it is almost impossible to behave rationally and theories based on the

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1 F.K. Knight [10]

hypothesis of rational behavior of every economic agent, therefore, lose their theoretical basis. Among others, a theory dealing with a decision problem under measurable uncertainty is known as 'expected utility theory'. However, the applicability of this theory, especially to problems of dynamic decisions, is rather restricted. In the real world, what happens in the future is very much influenced by our present decisions, most of which are essentially irrevocable; in other words, probabilistic distribution of future events is not independent from our present behavior, and may be said to be determined *historically*. Consequently we often know little about how our present decisions will effect the distribution of our future events. To assume that something is certain and known is, therefore, nothing but an assumption of perfect knowledge. For a study of decisions made in a historical setting, such as an investment decision in fixed capital, the assumption of rational behavior under a certain and known distribution of future events is almost meaningless.[2] Although the economics of measurable uncertainty has already been studied thoroughly and many implications for risk management have been derived from it, the study of true uncertainty is still lost in a thick mist, in spite of Professor G. L. S. Shackle's continuous efforts of nearly fifty years.

In the world of uncertainty, people cannot help but behave according to the basis of expectations. How are these expectations

2 Moreover, Hey [7] critisizes unequivocaly the subjective expected utility theory. He argues that the economics of uncertainty must abandon backward induction, therefore its pre-occupation with optimal rules of behavior, and concentrate instead on reasonable rules of thumb.

formed and what effect do they have on the workings of our economy? These problems have become to be thought of as the central theme of macrodynamics. The 'stagflation' of the early 70's caused the problem of expectations to be cast in quite a different light, that is the rational expectations theory. There is, however, no doubt of the inability of the rational expectations theory to deal with expectations under uncertainty in a Keynes/Knightian sense, for it still assumes that people are informed enough to be successful in forming their expectations. If our economy was something like a train running on tracks and rolling always from side to side, then although people would not know which way the train would roll next, they surely would know that they were, on the average, in the middle of the tracks as long as the train was not derailed. This being the case, then the rational expectations theory would provide us with a good description of our economy.[3]

It seems to me, however, that an airplane flying in the air is a much better metaphor of the real economy. Airplanes can stay in the air as long as they fly with enough speed to lift themselves up. Due to inertia, the flight of an airplane seems to be stable, but once it loses enough speed, it becomes apparent that there is nothing to sustain that airplane. In the real world, what gives our economy enough speed is, of course, vigorous investment activity of firms which results crucially from their expectations of future economic circumstances. Therefore, formation of expectations under true uncertainty is, as long as we share the same theoretical basis as

3 R. Bausor presented an exhaustive criticism on the rational expectations theory recently. See Bausor [2].

Keynes, one of the most important subjects for understanding economic fluctuations.

In spite of the importance of expectations, however, there have been few treatments of this subject in terms of macroscopic view. In this paper, confining the problem to a rather restricted scope, I would like to analyze the effect of a firm's demand expectations on the dynamic stability of the growth path in an oligopoly. In the course of the argument, I will also propose one possible explanation for stagflation. We will see that, when short-term expectations are relatively dominant, then the growth path is unstable and chronic stagflation may occur.

In the next section, I will give an algebraic expression to our model economy and present a simple expectations hypothesis, which consists of both short-term and long-term expectations in order to arrive at a tentative approximation of the problem. Stability properties will be examined in section III and some brief remarks will be given in the final section.

## II Model

The model economy where we will develop our argument is a simple Keynes/Kaleckian economy, which has no government sector and no foreign trade. There is only one commodity, which is used both as a consumption good and as an investment good. The price of this commodity is fixed in the short-run and equality of aggregate supply and demand is guaranteed by instantaneous quantity adjustment. The goods are produced by only one firm, which we call the representative oligopolistic firm (hereafter denoted as ROF).

According to *General Theory*, there exist two kinds of expectations formed by a firm, i.e. short-term and long-term expectations. Keynes observes that the former is concerned with price and output decisions and the latter with investment decisions. Although such expectations are thought to be the result of a variety of factors, many of which might have a highly qualitative nature, in this paper, we deal with them only in terms of the expected demand for a firm's products.

First, in the case of short-term demand expectations, we can suppose that these expectations are formed adaptively in response to the realized volume of demand. Chapter 5 of *General Theory* is very informative in respect to this point. There Keynes states[4]:

> But it will often be safe to omit express reference to *short-term* expectation, in view of the fact that in practice the process of revision of short-term expectation is a gradual and continuous one, carried on largely in the light of realized results; so that expected and realized results run into and overlap one another in their influence. For, although output and employment are determined by the producer's short-term expectations and not by past results, the most recent results usually play a predominant part in determining what these expectations are. It would be too complicated to work out the expectations *de novo* whenever a productive process was being started; and it would,

4 Keynes [9], Chapter 5, pp. 50–51.

> moreover, be a waste of time since a large part of the circumstances usually continue substantially unchanged from one day to the next. Accordingly it is sensible for producers to base their expectations on the assumption that the most recently realised results will continue, except in so far as there are definite reasons for expecting a change.

Although this passage does not necessarily mean that short-term expectations are always formed by inertia, we will, as a first approximation of the problem, suppose that the adaptive model can describe the real formation process of short-term expectations.[5]

From the above analysis, we can formulate the following equation as one which gives us a short-run formation process of demand expectations.

$$\dot{Q}^e = v(Q - Q^e), \quad v > 0 \tag{1}$$

where $Q^e$ is expected volume of demand for ROF's products and $Q$ is actual demand.

Suppose a fixed proportions technology and let $\sigma$ be the maximum ratio of output to capital, which is assumed constant, and $K$ be the amount of capital stock. The expected rate of capacity utilization, which we denoted by $u^e$, is;

$$u^e = Q^e/\sigma K \tag{2}$$

We may also assume that, when ROF forms short-term expectations, it neglects current changes in capital stock, so that the short-term expected rate of capacity utilization is affected only by changes in

5 A more general formulation with a time-varying adjustment coefficient is given in Lawson [13].

expected demand. Therefore, we can describe the behavior of the expected rate of capacity utilization over time by the following equation.

$$\dot{u}^e = \dot{Q}^e / \sigma K \qquad (3)$$

Substituting equations 2 and 3 into 1, we have

$$\dot{u}^e = v\,(u - u^e) \qquad (4)$$

where $u$ is the actual rate of capacity utilization, which is defined by $Q/\sigma K$.

Keynes introduced long-term expectations as the primary connective of the uncertainty-investment nexus, which has been regarded as the core of *General Theory* by many scholars. Before formulating our own hypothesis, let us take a look at Keynes' view on uncertainty and long-term expectations. In the following passage, which has been cited often,[6] Keynes states clearly that we have little information about the future that is needed in order to perform investments.[7]

> By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn.... The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970.

6 See Stohs [15] and Coddington [4] for example.

7 Keynes [10], pp. 113–14.

> *About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know* [my italics].

In such circumstances, how does the entrepreneur behave? Keynes indicates three possible techniques to manage uncertainty;[8]

> (1) We assume that the present is a much more serviceable guide to the future than a candid examination of past experience would show it to have been hitherto. In other words we largely ignore the prospect of future changes about the actual character of which we know nothing.
>
> (2) We assume that the *existing* state of opinion as expressed in prices and the character of existing output is based on a *correct* summing up of future prospects, so that we can accept it as such unless and until something new and relevant comes into the picture.
>
> (3) Knowing that our own individual judgment is worthless, we endeavour to fall back on the judgment of the rest of the world which is perhaps better informed. That is, we endeavour to conform with the behavior of the majority or the average. The psychology of a society of individuals each of whom is endeavouring to copy the others leads to what we may strictly term a *conventional* judgment.

What we can derive from these points is the conventional character of long-term expectations.[9] Such conventional behavior cannot

8 Keynes [10], p. 114.

9 He refers to conventional judgement also in *General Theory*. 'In practice we have tacitly agreed, as a rule, to fall back on what is, in truth, a convention. The essence of this convention... though it does not, of course, work out↗

be formulated neatly in a simple equation. Therefore, as many authors do,[10] we will treat long-term expectations exogenously in this paper, and these expectations are hereafter confined to the growth rate of demand. Although we assume the constancy of the growth rate for the simplicity of our exposition, 'this does not mean that we really believe that the existing state of affairs will continue indefinitely.'[11] In fact, what Keynes emphasizes is the extreme precariousness of conventional judgment. He states:[12]

> Now a practical theory of the future based on these three principles (cited above) has certain marked characteristics. In particular, being based on so flimsy a foundation, it is subject to sudden and violent changes. The practice of calmness and immobility, of certainty and security, suddenly breaks down. New fears and hopes will, without warning, take charge of human conduct. The forces of disillusion may suddenly impose a new conventional basis of valuation.

In contrast with short-term expectations we can point out the following two features for long-term expectations on demand. First, changes in productive capacity can no longer be neglected. Second, ROF may not form expectations about the absolute level of demand any longer, but it will form expectations about its trend growth rate. When the expected growth rate of demand is higher (lower)

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↘ quite so simply... lies in assuming that the existing state of affairs will continue indefinitely except in so far as we have specific reason to expect a change.' (p. 152).

10 See Adachi [1] and Eatwell [5] for example.

11 Keynes [9], Chapter 12, p. 152.

12 Keynes [10], p. 114. See also Keynes [9], pp. 149–50. Shackle called such flimsiness of expectations 'kaleidic factor'. See Shackle [14].

than the capacity growth rate, ROF will, under usual circumstances, expect that the rate of utilization will increase (decrease). From the long-run point of view, therefore, the following relationship may be expected by ROF.

$$\text{sign}(\dot{u}^e) = \text{sign}(\bar{q} - g) \tag{5}$$

where $\bar{q}$ denotes the growth rate of the long-term expected demand, which is here assumed to be a given constant, and $g$ is the capital accumulation rate $(\dot{K}/K)$, which is identical with the growth rate of capacity under the assumption of fixed proportions technology. We also assume that $\dot{u}^e$ is an increasing function of $(\bar{q} - g)$.

As already mentioned, Keynes assigns short-term expectations to price and output decisions and long-term expectations to investment decisions, but this cannot be said *a priori*. It has been often said that oligopolistic firms tend to determine their mark-ups in order to generate sufficient cash flow to finance their investment expenditure. In that case, we may expect that there exists a rather strong linkage between price setting and long-term expectations. On the contrary, investment may be influenced by short-term expectations because firms will partly delay (accelerate) intiations of their investment projects when they expect a recession (a boom) in the near future. Moreover, in the real economy, some firms make their decisions on the basis of short-term expectations, while some other firms base their decisions on long-term expectations. The representative firm must be supposed to be the 'average' of all those firms. Taking these points into account, we assume in this paper that the demand expectations of ROF are an amalgam of its short-term and long-term expectations. This amalgam hypothesis is formulated as follows.

$$\dot{u}^e = H(u - u^e, \bar{q} - g),\ H_1 \geq 0,\ H_2 \geq 0,\ H(0, 0) = 0 \qquad (6)$$

Hereafter, we will adopt a linear approximation of this equation to simplify algebraic manipulation. That is;

$$\dot{u}^e = h_1(u - u^e) + h_2(\bar{q} - g) \qquad (7)$$

where both $h_1$ and $h_2$ are non negative constants.

Next, we will introduce an investment function of the form

$$g = k(u^e - \bar{u}) + g_0 \qquad (8)$$

where $k$ and $\bar{u}$ are positive constants.[13] The second term of the right-hand side of this equation expresses the long-term desired rate of capital accumulation, which is, because of fixed proportions technology, assumed to be equal to the growth rate of the long-term expected demand. Equation 8 means that when the expected rate of capacity utilization exceeds its normal rate, which is denoted by $\bar{u}$, ROF accelerates the capital accumulation rate above the trend growth rate of its capital stock and *vice versa*.[14]

The classical saving assumption that wage income is entirely

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13 A similar investment function is adopted in Adachi [1].

14 Substituting epuation 8 into 7, we have

$$\dot{u}^e = h_1(u - u^e) - h_2k(u^e - \bar{u}) \qquad \text{(i)}$$

This equation can be approximated by the following difference equation.

$$u^e_t = u^e_{t-1} + h_1(u_{t-1} - u^e_{t-1}) - h_2k(u^e_{t-1} - \bar{u}) \qquad \text{(ii)}$$

Therefore, $u^e_t$ shows the Koyck lag distribution, that is

$$u^e_t = h_1 \sum_{i=1}^{\infty} (1 - h_1 - h_2k)^{i-1} u_{t-i} + h_2k\bar{u} \sum_{i=1}^{\infty} (1 - h_1 - h_2k)^{i-1} \qquad \text{(iii)}$$

Substituting (ii) into the investment function, we have

$$g_t = (1 - h_1 - h_2k) g_{t-1} + kh_1 u_{t-1} + (h_1 + h_2k) g_0 - kh_1\bar{u} \qquad \text{(iv)}$$

Since the last expression does not contain $u$ yet, our hypothesis can be applied to observable data.

spent on consumption and profits are all saved is assumed.[15] Then, in a closed economy without government economic activity, investment-saving equality is given by the Kaleckian formula, that is

$$I = P \tag{9}$$

where $I$ is nominal investment and $P$ total profits. The following equation is self-evident,

$$pQ = wN + P \tag{10}$$

where $p$ is the actual price level, $w$ the money wage rate and $N$ total labor input.

Substituting equation 10 into 9 and solving it with respect to $u$, we have

$$u = gn/\{\sigma(n - R)\} \tag{11}$$

where $R$ is the real wage rate $(w/p)$ and $n$ is output per worker, which is assumed to be given. We must, therefore, assume that both $g$ and $(n-R)$ are positive so that $u$ can be economically meaningful. These conditions are given by the following inequalities:

$$n > R \tag{12}$$

and

$$u^e > \bar{u} - g_0/k \tag{13}$$

Another decision in which demand expectations may be thought to play an important role is price setting. To make an explicit linkage between expectations and price, it is necessary to adopt what is called the target return pricing model, i.e. the mark-up rate is assumed to be set at a level which is high enough to yield a target

15 One of the major merits of adoption of the classical saving function is that it reflects the effect of real wage rate on the rate of capacity utilization through consumption demand.

rate of return on capital under a norml rate of utilization. An essential modification must, however, be made on this hypothesis in order to make the linkage. We must suppose that the mark-up rate is determined on the basis of the *expected* rate of capacity utilization.[16]

To realize the target rate of return under a certain level of expected demand, ROF must set the price of the product at the level given by the following equation,

$$\tilde{p} = \frac{\sigma u^e w}{n\,(\sigma u^e - \bar{r})} \qquad (14)$$

where $\tilde{p}$ is the target price level and $\bar{r}$ the target rate of return on capital. Here we will not argue extensively what determines the target rate of return, but we will suppose that the trend growth of capital is the major determinant.[17] Therefore, $\bar{r}=f(g_0)$, $f'>0$. For $\tilde{p}$ to be positive, $\sigma u^e$ must be larger than $\bar{r}$. We suppose next that the firm will increase (decrease) the actual price level when the target price is higher (lower) than that level. We have, therefore,

$$\dot{p} = a\,(\tilde{p} - p),\ a > 0 \qquad (15)$$

16 The behavior of a firm's price setting has an important effect on the stability property of the model.

17 It has been pointed by many Post-Keynesians that investment expenditure is the most important determinant of oligopolistic price. For example, P. Kenyon states: 'Post-Keynesians conclude—that oligopolistic firms, having some degree of discretionary power, set their margins above normal production costs, so that they can generate sufficient cash flow to finance from internal sources much of the investment expenditure they wish to undertake. That is, movements in prices depend upon the requirement of firm's for internally generated investment funds and upon movements in normal production costs. The mark-up is linked directly with the need to finance planned investment expenditure.' Kenyon [8], pp. 38–39. See also Eichner [5].

Equation 15 reflects price inertia and the short-run constancy of an oligopolistic price.

Next let $\hat{w}$ be the rate of increase of the nominal wage rate $\dot{w}/w$, for which we assume a given constant.[18] Differentiating $R$ with respect to time, we have

$$\frac{\dot{R}}{R}=\hat{w}-\frac{\dot{p}}{p} \tag{16}$$

The substitution of equation 14 and 15 into 16 yields

$$\frac{\dot{R}}{R}=\hat{w}-a\left\{\frac{\sigma u^{e}R}{n\,(\sigma u^{e}-\bar{r})}-1\right\} \tag{17}$$

Our model is now completed and the dynamic behavior of the

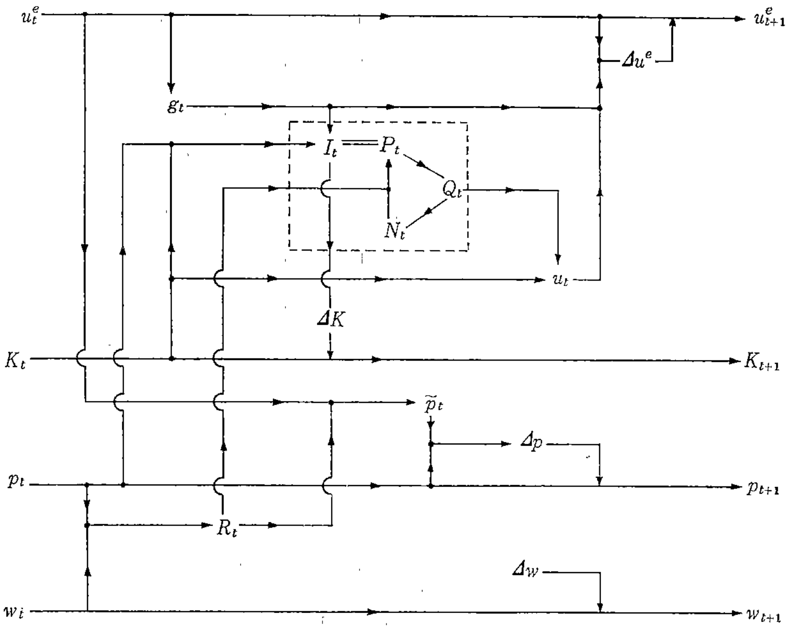


**Fig. 1** The Causal Relations Among the Variables

18 A possible alternative assumption is 'quasi perfect expectations of inflation', i. e. $\hat{w}=m(\dot{p}/p)$, where $m$ is a positive constant. Adoption of this assumption will yield an interesting result.

model is described by equations 7, 8, 11 and 17, which contain four unknowns; $u^e$, $u$, $g$ and $R$.

## III The stability of the growth path

In this section we will examine the local stability of the stationary point for two extreme cases in order to simplify our argument.

Case A ($h_1>0$ and $h_2=0$): Long-term expectations are completely excluded from ROF's decision making in this case, and we may think of such a condition as a purely myopic case. The stationary point is, then, unique and given by the following two equations.

$$u^{e*}=\frac{(\hat{w}+a)\,\bar{r}+a\,(k\bar{u}-\bar{q})}{\sigma k+\hat{w}a} \tag{18}$$

$$R^*=\frac{n\,(\hat{w}+a)\,\{k\,(\sigma\bar{u}-\bar{r})+\sigma\,(\bar{r}-\bar{q})\}}{\sigma\,\{\bar{r}\,(\hat{w}+a)+a\,(k\bar{u}-\bar{q})\}} \tag{19}$$

The dynamic behavior of this system is given in **Fig. 2.**[19] As is shown in the figure, the stationary point is a saddle point and any dynamic path, except two special cases, does not converge to ($u^{e*}$, $R^*$).[20] The set of $u^e$ and $R$ which keeps the price level constant over time is given by the broken line in the diagram.[21] In the area above (below) this line, the price level increases (decreases) over time.

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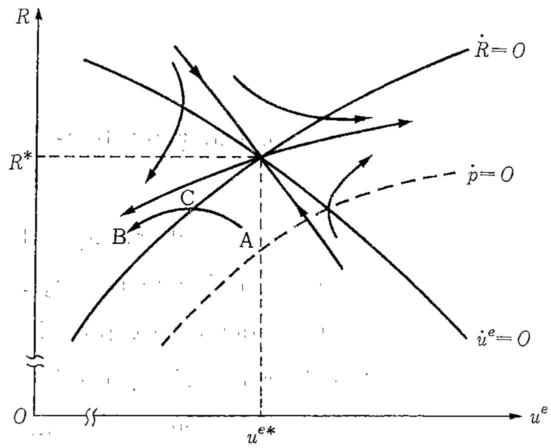
19 We cannot specify the sign of the slope of the line $\dot{u}^e=0$, but this does not disturb the nature of the figure.

20 See Appendix.

21 The algebraic expression of this curve is as follows.

$$R=\frac{n\,(\sigma u^e-\bar{r})}{\sigma u^e}$$




(**Fig. 2**)

If an initial condition is given at a point like A in the figure, a stagflation process will emerge. On the path towards point B in the figure, the actual rate of capacity utilization is constantly lower than the expected level, so that the expectations are adjusted downwards, while the actual price is increased over time so as to realize the target return on capital. Differentiating $\dot{p}/p$ with respect to time, we have

$$\frac{d}{dt}\left(\frac{\dot{p}}{p}\right)=\frac{a\sigma u^e R}{n\,(\sigma u^e-\bar{r})}\left\{\frac{\dot{R}}{R}-\frac{\bar{r}}{\sigma u^e-\bar{r}}\cdot\frac{\dot{u}^e}{u^e}\right\} \tag{20}$$

Before reaching point C, therefore, the rate of inflation is certainly increased because the right hand side of equation 20 is positive when $\dot{R}$ is positive and $\dot{u}^e$ is negative. Accelerating inflation will catch up with the wage increment rate at point C, and after that the real wage rate will be decreased by inflation. A decline of the real wage rate has a negative effect on the demand for the consumption goods. Investment will decelerate because of the low rate of capacity utilization. Therefore the relative shortage of aggregate demand is increased and the actual utilization rate will be decreased. Such a situation will compel the firm to slow down the accumulation rate and to raise the price level still more. This means that the stagflation reproduces itself. A firm's attempt to increase profit by raising the price level may succeed if the others do not follow. However, when almost all firms in the economy try to do so, the aggregate real demand will fall and profit may decrease.

Case B ($h_1=0$ and $h_2>0$): In this case, short-term demand expectations are completely neglected. ROF expects that the demand will grow at rate $\bar{q}$ irrespective of the currently realized demand.

The stationary point, which is defined by the following two equations, can be easily shown as a stable node.[22]

$$u^{e*}=\bar{u} \tag{21}$$

$$R^*=\left(1+\frac{\hat{w}}{a}\right)\frac{n(\sigma\bar{u}-\bar{r})}{\sigma\bar{u}} \tag{22}$$

At the stationary point $u^{e*}$ equals $\bar{u}$, yet this does not necessarily mean that the actual rate of utilization $u^*$ equals $\bar{u}$. In fact,

$$u^*=\frac{n\bar{q}}{\sigma(n-R^*)}=\frac{a\bar{q}\bar{u}}{(a+\hat{w})\bar{r}-\hat{w}\sigma\bar{u}} \tag{23}$$

so that $u^*=\bar{u}$ only if the parameters appearing in the above equation satisfy a certain relationship.

Suppose the situation that $u^*<\bar{u}$ over a long period. ROF will, then, infer that the normal rate of capacity utilization is set too high and will think that operation of capacity at that level is im-

22 See Appendix.

possible over the long-run. Therefore, ROF will revise either the long-term rate of capital accumulation $g_0$ or the normal rate of the utilization $\bar{u}$ downwards so that the normal operation of the equipment can be realized in the long-run. Downwards revision in $g_0$ will decrease the gap between $u^*$ and $\bar{u}$, because

$$\frac{\partial u^*}{\partial g_0}\bigg|_{g_0=\bar{q}} = \frac{-a\bar{q}\,(a+\hat{w})\,\{f(\bar{q})+\bar{u}kf'(\bar{q})\}}{k\,\{(a+\hat{w})f(\bar{q})-\sigma\bar{u}\hat{w}\}^2} < 0 \qquad (24)$$

Therefore, if this adjustment continues for a sufficient length of time, $\bar{u}$ will converge to $u^*$. However, when ROF tries to adjust $\bar{u}$ to $u^*$, the gap will be increased. Differentiating $u^*/\bar{u}$ with respect to $\bar{u}$, we have

$$\frac{\partial(u^*/\bar{u})}{\partial\bar{u}} = \frac{a\hat{w}\sigma\bar{q}}{\{(a+\hat{w})\,\bar{r}-\hat{w}\sigma\bar{u}\}^2} > 0 \qquad (25)$$

Therefore, in an economy where $\bar{u}$ tends to be determined according to $u^*$, another kind of instability may be expected in the long-run. In the real world, however, both parameters will be adjusted and their compound effect on $u^*$ depends on which is subject to larger adjustments.

## IV Concluding remarks

In the previous section we investgated the stability properties of demand expectations. We have only examined two special cases, but, in reality, both $h_1$ and $h_2$ are thought to be positive. Although no investigation for this general case has been presented, it may safely be assumed that any growth path is unstable (stable) when short-term (long-term) expectations are relatively dominant.[23]

However, since the trend growth $\bar{q}$, which is assumed to represent long-term expectations in the present paper, stands on the basis of conventional judgment which Keynes thought extremely precarious, this conclusion does not necessarily mean that our economy can grow steadly along a trend growth path under the dominance of long-term expectations. We can only say that there is no endogenous cumulative process of disequilibrium when short-term expectations have little or no effect on a firm's decision.

There still remains an important question; under what circumstances does one type of expectations become dominant over the other? When long-term expectations are formed with great confidence, short-run fluctuations of demand will be neglected and excluded from a firm's decision and this stabilizes the growth path. Such a state of confidence is, as J. Eatwell argues in his recent paper, thought to depend very much upon the stability of the institutional environment.[24] However, to formulate such dependence within a simple model is, needless to say, impossible. Hence we have to argue this problem on a different level of abstraction from what we have just done. However it is an issue of another day.

## Appendix

Our dynamic system given in section II can be reduced to the following two differential equations.

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23 The stability effect of long-term expectations has been realized by many authors. For example, See Adachi [1] and Eatwell [5].

24 Eatwell [5].

$$\dot{u}^e = h_1\left[\frac{n\{k(u^e-\bar{u})+\bar{q}\}}{\sigma(n-R)} - u^e\right] - h_2 k(u^e-\bar{u}) \qquad \text{(A-1)}$$

$$\dot{R} = R\left[\hat{w} - a\left\{\frac{\sigma u^e R}{n(\sigma u^e - \bar{r})} - 1\right\}\right] \qquad \text{(A-2)}$$

Let $D$, domain of our model, be a subset of $R_+^2$ and define it as follows.

$$D = \left\{(u^e, R) \,\middle|\, u^e > \max(\bar{u} - \bar{q}/k,\ \bar{r}/\sigma),\ R < n\right\} \qquad \text{(A-3)}$$

We also assume that the stationary point of the system, which we denote by ($u^{e*}$, $R^*$) is economically feasible, i. e. ($u^{e*}$, $R^*$) $\in$D.

For case A, the coefficient matrix of the linearized system evaluated at the stationary point is as follows.

$$M_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad \text{(A-4)}$$

where

$$A_{11} = \frac{h_1(ka+\hat{w}\sigma)(k\bar{u}-\bar{q})}{k(\hat{w}+a)\bar{r} - \hat{w}\sigma(k\bar{u}-\bar{q})} \qquad \text{(A-4-a)}$$

$$A_{12} = \frac{h_1\sigma\{(\hat{w}+a)\bar{r}+a(k\bar{u}-\bar{q})\}^2}{n(ak+\hat{w}\sigma)\{k(\hat{w}+a)\bar{r}-\hat{w}\sigma(k\bar{u}-\bar{q})\}} \qquad \text{(A-4-b)}$$

$$A_{21} = \frac{n(\hat{w}+a)^2(ak+\hat{w}\sigma)^2\bar{r}}{a\sigma\{(\hat{w}+a)\bar{r}+a(k\bar{u}-\bar{q})\}^2} \qquad \text{(A-4-c)}$$

$$A_{22} = -(\hat{w}+a) \qquad \text{(A-4-d)}$$

From equation 18, $\bar{r}(\hat{w}+a)+a(k\bar{u}-\bar{q})$ must be positive so that $u^{e*}$ can be positive. Under the assumption that ($u^{e*}$, $R^*$) $\in D$, the following inequality holds.

$$n - R^* = n\frac{k(\hat{w}+a)\bar{r}-\hat{w}\sigma(k\bar{u}-\bar{q})}{\sigma\{\bar{r}(\hat{w}+a)+a(k\bar{u}-\bar{q})\}} > 0 \qquad \text{(A-5)}$$

From this condition, it can be directly shown that

$$\det M_1 = -\frac{h_1(\hat{w}+a)(ka+\hat{w}\sigma)\{(\hat{w}+a)\bar{r}+a(k\bar{u}-\bar{q})\}}{a\{k(\hat{w}+a)\bar{r}-\hat{w}\sigma(k\bar{u}-\bar{q})\}} < 0 \qquad \text{(A-6)}$$

The stationary point is, therefore, proved to be a saddle point.

For case B, the coefficient matrix of the linearized system at ($u^{e*}$, $R^*$) is as follows.

$$M_2 = \begin{bmatrix} -h_2 k & 0 \\ \dfrac{n(a+\hat{w})^2\bar{r}}{a\sigma\bar{u}^2} & -(a+\hat{w}) \end{bmatrix} \qquad \text{(A-7)}$$

Hence, we have the following relationships.

$$\text{tr } M_2 = -h_2 k - (a+\hat{w}) < 0 \qquad \text{(A-8)}$$

$$\det M_2 = h_2 k(a+\hat{w}) > 0 \qquad \text{(A-9)}$$

Therefore,

$$(\text{tr } M_2)^2 - 4 \det M_2 = \{h_2k - (a + \hat{w})\}^2 > 0 \qquad \text{(A-10)}$$

From these three equations, we find that the stationary point is a stable node.

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