Sequential Investment in Pollution Control Equipment under Uncertainty*

Motoh Tsujimura†

Abstract

In this paper, we investigate investment in pollution control capital under uncertainty. We assume that a firm's output generates pollution as a by-product, which reduces the productivity of capital. The dynamics of pollution are assumed to be governed by a stochastic differential equation. Thus, the firm must incur the cost of investing in pollution control capital to reduce the pollutant. The firm also pays an environmental tax, which is proportional to its emissions. We assume that the firm can invest as necessary. Hence, the firm's problem is to choose its investment timing under uncertainty. This problem is formulated as a singular stochastic control problem. We solve the firm's problem by using variational inequalities. The optimal investment strategy is characterized by a threshold for investing in pollution control equipment. We also conduct comparative static analysis of the model's parameters. We find that an increase in the volatility of abatement capacity discourages investment in pollutant abatement, whereas an increase in the environmental tax rate encourages such investment.

Keywords : pollution control equipment, singular stochastic control, variational inequalities

1 Introduction

There has been an increase in the demand for energy as the global economy has grown. According to the World Bank[1], in 2010, global gross domestic product was USD 41,428 billion, 1.28 times greater than in 2000, when it was USD 32,334 billion (both in constant 2000 prices). World energy use in 2010 was 12,324 megatons, 1.26 times greater than in 2000, when it was 9,802 megatons (both in oil-equivalent megatons). A by-product of this increase in energy use is more pollutants such as CO_2, SO_2, and NO_x. These pollutants generate air pollution in the form of climate change, acid rain, and photochemical smog as external effects. These pollutants must be controlled appropriately Perman et al. (2003).

Pollutant abatement investment is needed to control pollutants. For example, coal-fired power plants emit air pollutants. Many countries regulate these pollutant emissions ; for

*This research was supported in part by a Grant-in-Aid for Scientific Research (No.24510213) from the Japan Society for the Promotion of Science.

†Address : Karasuma-Higashi-iru, Imadegawa-dori, Kamigyo-ku, Kyoto, 602-8580 JAPAN
Phone : +81−75−251−4582 ; E-mail : mtsujimu@mail.doshisha.ac.jp

1 We use data from World Bank's Web site (http : //data. worldbank. org/).

example, the U.S. has the Clean Air Act. Equipment to remove SO_2 and NOx has been installed in coal-fired power plants, and old plants have been replaced by more-efficient power plants. In this paper, we explore how a firm invests in relatively small pieces of equipment such as pollutant removal equipment.

There is much research on pollutant abatement investment problems. For example, Pindyck (2000) explores the conditions under which policymakers implement pollutant emissions reduction policies under the uncertainty arising from general economic conditions. He shows that the optimal pollutant reduction policy is characterized by a threshold. When the level of economic uncertainty reaches this threshold, the policymaker implements the pollutant reduction policy. Lin et al. (2007) also investigate a similar problem. These studies ignore environmental taxes, which influence investment decisions. Farzin and Kort (2000) investigate the amount firms invest in pollutant abatement capital when there is uncertainty about pollution taxes. They investigate how changes to the pollution tax rate affect investment in pollutant abatement capital. Zhao (2003) investigates how abatement cost uncertainties affect firms' abatement investment incentives, and finds that emissions trading helps maintain firms' incentives to invest in pollutant abatement. Pindyck (2000) extends his model to incorporate ecological uncertainty, which is represented by dynamics in the stock of pollution. Saltari and Travaglini (2011) examine a green firm's abatement capital investment problem under ecological uncertainty. They show that there are two optimal abatement investment regimes : one in which there is positive investment, and one in which there is none.

We investigate a firm's pollutant abatement investment problem under uncertainty. As does Farzin and Kort (2000), we assume that the firm is risk neutral and produces a single output that it sells in a competitive market. The production process generates pollution emissions that are proportional to output. The firm must pay a pollution tax per unit of emissions ; unlike Farzin and Kort (2000), we assume that this tax rate is constant. Thus, the firm has an incentive to invest in pollutant control equipment to maximize profit. Following Pham (2006), we assume that the stock of pollutant abatement capacity is governed by a stochastic differential equation. In addition, we consider the case in which the firm can invest in abatement capital as necessary. This makes our analysis differ from previous research. The investment problem that we model is classified as a partially irreversible investment problem (see, for example, Abel and Eberly (1996), Guo and Pham (2005), Pham (2006), and Merhi and Zervos (2007)). We formulate the firm's problem as a singular stochastic control problem, which we solve by using variational inequalities. The optimal investment strategy is characterized by a threshold for investment in pollution control equipment. Having conducted comparative static analysis, we find that an increase in the volatility of abatement capacity discourages pollutant abatement investment, whereas an increase in the environmental tax rate encourages such investment.

The rest of the paper is organized as follows. In Section 2, we describe the setup of the firm's problem. In Section 3, we solve the firm's problem. In Section 4, we present a numerical analysis. Section 5 concludes the paper.

2 The Model

Suppose that a firm produces a single output by using a variable input L such as labor and sells its output in a competitive market. The input price $w>0$ and the output price $p>0$ are assumed to be constants. The firm's production function $F(L_t)$ has the form :

$$F(L_t)=aL_t^{\gamma}, \tag{2. 1}$$

where $a>0$ is a constant that reflects the level of production technology and $\gamma\in(0,1)$ is the output elasticity of the variable input. The production process generates pollution emissions E proportional to output. Pollution emission E is given by :

$$E_t=\eta(K_t)F(L_t), \tag{2. 2}$$

where η is the emission coefficient function of the stock of abatement capacity K. Following Farzin and Kort (2000), we assume that $\eta(K)>0$, $\eta'(K)<0$, and $\eta''(K)>0$. We specify η as :

$$\eta(K_t)=bK_t^{-\lambda}, \tag{2. 3}$$

where $b>0$ is a constant conversion factor between output and pollutant emission and $\lambda>0$ is the emission abatement elasticity of abatement capacity. The firm must pay a tax $\tau>0$ per unit of emissions, which is assumed to be constant. The firm invests in abatement capacity to reduce pollution emissions. Let I_t be cumulative purchases of abatement equipment up to time t. The firm can purchase the abatement equipment at any time t at a constant price of $c>0$. The process of abatement investment is left-hand-limit-adapted, nonnegative, and nondecreasing, with $I_{0-}=0$. Following Pham (2006), we assume that the firm's abatement capacity evolves according to :

$$\mathrm{d}K_t = -\delta K_t \mathrm{d}t + \sigma K_t \mathrm{d}W_t + \mathrm{d}I_t, \quad K_{0-} = k \ (>0), \tag{2. 4}$$

where $\delta \in (0,1)$ is the constant depreciation rate of abatement equipment and $\sigma > 0$ represents the volatility of abatement capacity. W_t is a standard Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}_{t\geq 0})$, where $\mathcal{F}_t$ is generated by W_t in $\mathbb{R}$; that is, $\mathcal{F}_t = \sigma(W_s, s \leq t)$.

The firm's operating profit $\tilde{\pi}$ at time t is given by :

$$\tilde{\pi}(K_t) = pF(L_t) - wL_t - \tau\eta(K_t)F(L_t). \tag{2. 5}$$

We assume that the variable input can be adjusted instantaneously and at no cost. Hence, the firm's maximized instantaneous operating profit $\pi(K_t)$ at time t is :

$$\pi(K_t) = (p - \tau b K_t^{-\lambda})^{\alpha} h, \tag{2. 6}$$

where $\alpha := 1/(1-\gamma)$ (>1) and $h := \alpha^{\alpha}(\alpha-1)^{-\alpha-1} w^{1-\alpha} \mathrm{a}^{-\alpha}$ (>0). The firm's expected discounted profit $J(k\,;I)$ is given by :

$$J(k\,;I) = \mathbb{E}\left[\int_0^{\infty} \mathrm{e}^{-rt}\pi(K_t)\mathrm{d}t - c\int_0^{\infty} \mathrm{e}^{-rt}\mathrm{d}I_t\right], \tag{2. 7}$$

where $r > 0$ is the discount rate, $I = \{I_t\}_{t\geq 0} \in \mathcal{A}$ denotes the investment strategy, and $\mathcal{A}$ is the set of all admissible investment strategies. In this context, it is assumed that :

$$\mathbb{E}\left[\int_0^{\infty} \mathrm{e}^{-rt}\pi(K_t)\mathrm{d}t\right] < \infty \tag{2. 8}$$

and :

$$\mathbb{E}\left[\int_0^{\infty} \mathrm{e}^{-rt}\mathrm{d}I_t\right] < \infty. \tag{2. 9}$$

Therefore, the firm's problem is to maximize its expected discounted profit over $\mathcal{A}$:

$$V(k) = \sup_{I \in \mathcal{A}} J(k\,;I) = J(k\,;I^*), \tag{2. 10}$$

where V is the value function and I^* is the optimal investment strategy.

3 Variational Inequalities

From the formulation of the firm's problem (2. 10), one would expect the firm, under its optimal investment strategy, to invest in pollutant abatement capacity whenever the stock of abatement capacity K falls below a threshold $\underline{k}$. To verify this conjecture, we solve the firm's problem (2. 10) by using variational inequalities.

The variational inequalities of the agent's problem (2. 10) are as follows :

Definition 3. 1 (Variational Inequalities). *The following relations are called the variational inequalities for the agent's problem* (*2. 10*) *:*

$$\mathcal{L}V(k)+\pi(k)\leq 0, \tag{3. 1}$$

$$V'(k)\leq c, \tag{3. 2}$$

$$[\mathcal{L}V(k)+\pi(k)][V'(k)-c]=0, \tag{3. 3}$$

where the operator $\mathcal{L}$ *is defined by :*

$$\mathcal{L}:=\frac{1}{2}\sigma^2k^2\frac{\mathrm{d}^2}{\mathrm{d}k^2}-\delta k\frac{\mathrm{d}}{\mathrm{d}k}-r. \tag{3. 4}$$

See, for example, Harrison and Taksar (1983) and Merhi and Zervos (2007) for derivation of the variational inequalities. The variational inequalities can be summarized as follows :

$$\max[\mathcal{L}V(k)+\pi(k),\ V'(k)-c]=0. \tag{3. 5}$$

Let H be the continuation region given by :

$$H:=\{k\,;V'(k)<c\}. \tag{3. 6}$$

Consider the well-known Skorohod Lemma, which is proven by, for example, Rogers and Williams (2000, pp.117–118).

Lemma 3. 1. *For any* $k>0$, *and given a boundary* $\underline{k}>0$, *there exists a unique adapted cadlag process* $K^*=\{K^*_t\}_{t\geq 0}$ *and a nondecreasing process* I^* *that satisfy the following Skorohod problem :*

$$\mathrm{d}K^*_t = -\delta K^*_t \mathrm{d}t + \sigma K^*_t \mathrm{d}W_t + \mathrm{d}I_t, \quad K^*_{0-} = k,\ t \geq 0, \tag{3. 7}$$

$$K^*_t \in [\underline{k}, \infty) \quad a.e.,\ t \geq 0, \tag{3. 8}$$

$$\int_0^t \mathbf{1}_{\{K^*_s > \underline{k}\}} \mathrm{d}I^*_s = 0. \tag{3. 9}$$

Furthermore, if $k \geq \underline{k}$, *then* I^* *is continuous. If* $k<\underline{k}$, *then* $I^*_0 = \underline{k} - k$ *and* $K^*_0 = \underline{k}$.

The Skorohod Lemma implies that K^* is a reflected diffusion at the boundary $\underline{k}$ and I^* is the local time of K^* at $\underline{k}$. Condition (3. 9) implies that I^* increases only when K^* reaches $\underline{k}$. Then, the continuation region H becomes :

$$H = \{k\ ;\ k>\underline{k}\}. \tag{3. 10}$$

Let $\phi \in C^2$ be a function and let $T<\infty$ be a stopping time. From Ito's formula for cadlag semimartingales, we have :

$$\begin{aligned} \mathrm{e}^{-rT}\phi(K_T) = \phi(k) + \int_0^T \mathrm{e}^{-rT}\mathcal{L}\phi(K_t)dt + \int_0^T \mathrm{e}^{-rT}\sigma K_t \phi'(K_t)dW_t \\ + \int_0^T \mathrm{e}^{-rT}\phi'(K_t)\mathrm{d}I_t^c + \sum_{0\leq t\leq T} \mathrm{e}^{-rT}[\phi(K_t) - \phi(K_{t-})]. \end{aligned} \tag{3. 11}$$

We can now prove that a solution to the variational inequalities is optimal. The following theorem is the well-known verification theorem. In Appendix A, we prove the theorem by following Pham (2006, Proposition 1. 3. 1) and Yang and Liu (2004, Theorem 1).

Theorem 3. 1. (I) *Let* ϕ *be a solution of the variational inequalities that satisfies the following :*

$$\lim_{t\to\infty} \mathrm{e}^{-rt}\phi(K_t) = 0. \tag{3. 12}$$

Then, we obtain :

$$\phi(k) \geq V(k), \quad k > 0. \tag{3. 13}$$

(*II*) ϕ *also satisfies the following :*

$$\mathcal{L}\phi(y) + \pi(k) = 0, \quad k > \underline{k}, \tag{3. 14}$$

$$\phi(k) = c(\underline{k} - k) + d, \quad k \leq \underline{k}, \tag{3. 15}$$

where d is constant. Then, there exists an optimal policy $I^* \in \mathcal{A}$ *such that :*

$$\phi(k) = V(k). \tag{3. 16}$$

That is, ϕ *is the value function and* I^* *is the corresponding optimal policy.*

Proof. See Appendix A.

In what follows, for analytical simplicity, we assume that $\gamma = 1/2$. This yields $\alpha = 2$. For $k > \underline{k}$, the variational inequalities (3. 1)−(3. 3) lead to the following ordinary differential equation :

$$\frac{1}{2}\sigma^2 k^2 \phi''(k) - \delta k \phi'(k) - r\phi(k) + \pi(k) = 0. \tag{3. 17}$$

The general solution of the ordinary differential equation (3. 17) with $\pi(k) = 0$ is given by :

$$\phi(k) = A_1 k^{\beta_1} + A_2 k^{\beta_2}, \quad k > \underline{k}, \tag{3. 18}$$

where A_1 and A_2 are constants to be determined. β_1 and β_2 are the solutions to the following characteristic equation :

$$\Gamma(\beta) := \frac{1}{2}\sigma^2 \beta(\beta - 1) - \delta\beta - r = 0. \tag{3. 19}$$

These solutions are given by :

$$\beta_1=\frac{1}{2}+\frac{\delta}{\sigma^2}+\left[\left(\frac{1}{2}+\frac{\delta}{\sigma^2}\right)^2+\frac{2r}{\sigma_2}\right]^{\frac{1}{2}}>1,\ \beta_2=\frac{1}{2}+\frac{\delta}{\sigma^2}-\left[\left(\frac{1}{2}+\frac{\delta}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}\right]^{\frac{1}{2}}<0. \tag{3. 20}$$

To find a particular solution of (3. 17) we try a function of the form $\phi_p(k)=B_1p^2h-2B_2\tau bpk^{-1}h+B_3\tau^2b^2k^{-2}h$. Given that $\mathcal{L}\phi_p(k)+\pi(k)=0$, we have :

$$B_1=\frac{1}{r},\quad B_2=-\frac{1}{\rho_1},\quad B_3=\frac{1}{\rho_2}, \tag{3. 21}$$

where $\rho_1:=r-\delta\lambda-0.5\lambda(\lambda+1)\sigma^2$ and $\rho_2:=r-2\delta\lambda-\lambda(2\lambda+1)\sigma^2$. It follows from assumption (2. 8) that $\rho_1>0$ and $\rho_2>0$. The general solution of (3. 17) is :

$$\phi(k)=A_1k^{\beta_1}+A_2k^{\beta_2}+\frac{p^2h}{r}-\frac{2p\tau bk^{-\lambda}h}{\rho_1}+\frac{\tau^2b^2k^{-2\lambda}h}{\rho_2},\quad k>\underline{k}. \tag{3. 22}$$

Because there is no upper threshold and $\beta_1>0$, we set $A_1=0$ to prevent the value function from going to infinity. Then, the general solution to (3. 17) is :

$$\phi(k)=A_2k^{\beta_2}+\frac{p^2h}{r}-\frac{2p\tau bk^{-\lambda}h}{\rho_1}+\frac{\tau^2b^2k^{-2\lambda}h}{\rho_2},\quad k>\underline{k}. \tag{3. 23}$$

The second, third, and fourth terms on the right-hand side of (3. 23) represent the expected present value of the firm's profit when it does not invest in the abatement capital forever :

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\pi(K_t)dt\right]=\frac{p^2h}{r}-\frac{2p\tau bk^{-\lambda}h}{\rho_1}+\frac{\tau^2b^2k^{-2\lambda}h}{\rho_2}. \tag{3. 24}$$

It follows from the definition of the firm's problem that the function ϕ satisfies the following inequality :

$$\phi(k)>\frac{p^2h}{r}-\frac{2p\tau bk^{-\lambda}h}{\rho_1}+\frac{\tau^2b^2k^{-2\lambda}h}{\rho_2}. \tag{3. 25}$$

This implies that $A_2>0$.

Let ϕ be redefined as the following candidate function of the value function :

$$\phi(k)=\begin{cases}\psi(k):=A_2k^{\beta_2}+\dfrac{p^2h}{r}-\dfrac{2p\tau bk^{-\lambda}h}{\rho_1}+\dfrac{\tau^2b^2k^{-2\lambda}h}{\rho_2}, & k>\underline{k},\\ \psi(\underline{k})-c(\underline{k}-k), & k\leq k.\end{cases} \tag{3. 26}$$

The two unknowns A_2 and $\underline{k}$ are determined by the following simultaneous equations :

$$\psi'(\underline{k}) = c, \tag{3. 27}$$

$$\psi''(\underline{k}) = 0. \tag{3. 28}$$

Equation (3. 27) is the smooth-pasting condition and (3. 28) is the super contact condition (see Dumas (1991) for details). In the next section, we numerically solve these simultaneous equations.

4 Numerical Analysis

In this section, we numerically calculate the threshold $\underline{k}$ and investigate the effects of changes in the parameters on the threshold. The basic parameter values are : $r = 0.1$, $a = 1$, $\delta = 0.03$, $\sigma = 0.08$, $\gamma = 0.5$, $\lambda = 1$, $w = 1$, $b = 0.5$, $c = 1$, $p = 1$, and $\tau = 0.1$. From these values, we obtain $A_2 = 2.52248$ and $\underline{k} = 1.59834$.

Figures 1–5 illustrate the comparative static effects on the threshold k. Figure 1 shows that the continuation region H is increasing in the volatility of abatement capacity σ. This result implies that the incentive to wait for new information about abatement capacity becomes stronger as uncertainty about future abatement capacity increases. This result is consistent with the standard results from real options analysis.

Figure 2 shows that the continuation region H is decreasing in the emission abatement elasticity of abatement capacity λ. This result is contrary to expectations. Because the emission coefficient function η is decreasing in λ when abatement capacity exceeds one ($k >$

Figure 1　Comparative static effect of σ on the threshold.

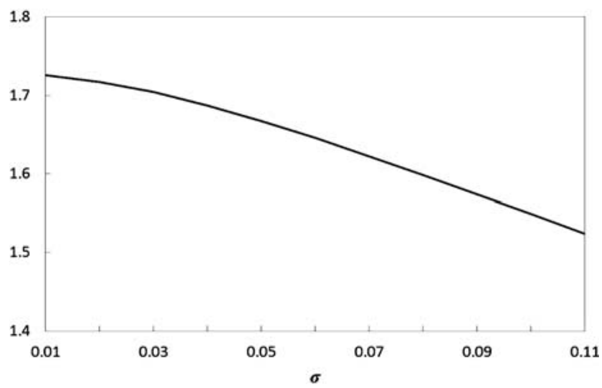


Figure 2 Comparative static effect of λ on the threshold.

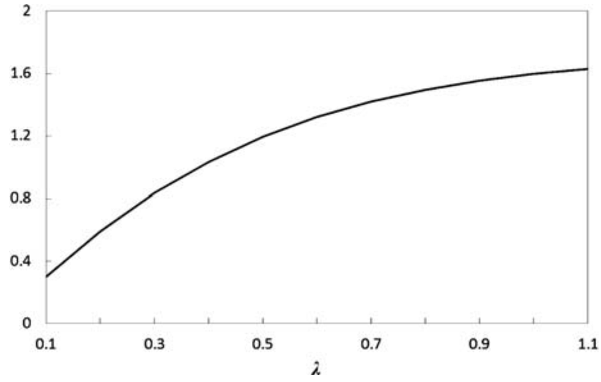


Figure 3 Comparative static effect of τ on the threshold.

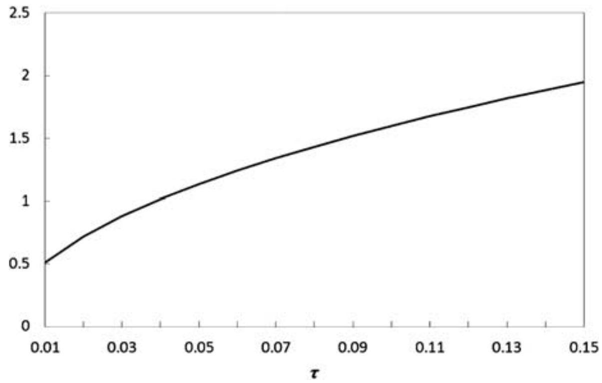
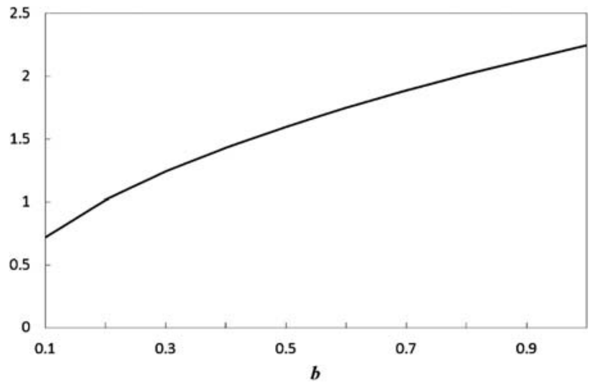


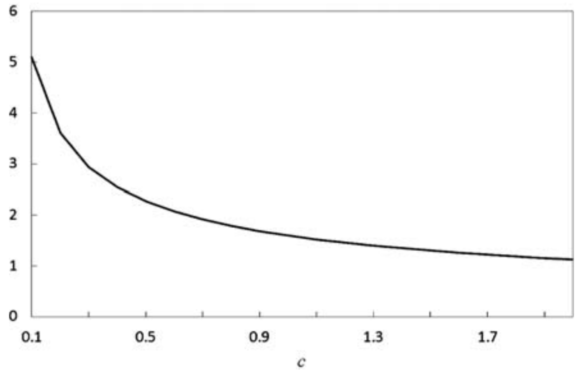
Figure 4 Comparative static effect of b on the threshold.



1), the pollutant emission function E is decreasing in λ. Our base case numerical result implies that it is reasonable to assume that the initial level of abatement capacity level exceeds one in our setting.

Figure 3 illustrates that the continuation region H is decreasing in the environmental tax

Figure 5 Comparative static effect of c on the threshold.



rate τ. The firm pays more tax as the tax rate increases. Hence, the firm has an incentive to invest in abatement capacity to avoid paying more tax, which reduces profits.

Figure 4 shows that the continuation region H is decreasing in the conversion factor b. The higher b, the more the pollutant is emitted. Hence, the firm increases its investment in its abatement capacity.

Figure 5 shows that the continuation region H is increasing in the price of abatement equipment c. When the price of abatement equipment rises, the firm incurs more abatement cost. Hence, the firm postpones its investment in abatement capacity.

These results provide useful insights into investment decisions under uncertainty.

5 Conclusion

In this paper, we examined the firm's pollutant abatement investment problem under uncertainty. We formulated the problem as a singular stochastic control problem and used variational inequalities to solve it. We showed that the optimal investment strategy is characterized by a threshold. That is, the firm invests in abatement capacity when its stock falls below the threshold. In addition, we conducted comparative static analysis of the model's parameters. We found that an increase in the volatility of abatement capacity discourages pollutant abatement investment, whereas an increase in the environmental tax rate encourages such investment.

To conclude the paper, we suggest possible extensions to our model. It is worth exploring the mechanism through which changes in the emission abatement elasticity of abatement capacity affects the threshold. It is also necessary to examine the effect of output prices. This could be achieved by using a stochastic differential equation to model the output price. Our framework could also be used to examine specific pollutants and/or pollutant reduction projects. We leave these important topics to future research.

Appendix A.

Proof of theorem 3. 1. (I) For $I\in\mathcal{A}$, let $T_n=\inf\{t\geq 0;\ K_t\geq n\}\wedge n,\ n\in\mathbb{N}$ be the finite stopping time. We apply (3. 10) between $t=0$ and $t=T_n$ and take expectations. We obtain :

$$\mathbb{E}[e^{-rT_n}\phi(K_{T_n})]=\phi(k)+\mathbb{E}\left[\int_0^{T_n}e^{-rt}\mathcal{L}\phi(K_t)dt\right]+\mathbb{E}\left[\int_0^{T_n}e^{-rt}\phi'(K_t)dI_t^c\right]$$
$$+\mathbb{E}\left[\sum_{0\leq t\leq T_n}e^{-rt}[\phi(K_t)-\phi(K_{t-})]\right]. \tag{A. 1}$$

Because (3. 6) and $K_t-K_{t-}=\Delta I_t$, the mean-value theorem implies that :

$$\phi(K_t)-\phi(K_{t-})=\phi(\theta)\Delta I_t\geq c\Delta I_t, \tag{A. 2}$$

where $\theta\in(K_{t-},\ K_t)$. It follows from (3. 5) and (3. 6) that (A. 1) can be rewritten as :

$$\mathbb{E}[e^{-rT_n}\phi(K_{T_n})]\geq\phi(k)-\mathbb{E}\left[\int_0^{T_n}e^{-rt}\pi(K_t)dt\right]-\mathbb{E}\left[\int_0^{T_n}e^{-rt}c\,dI_t^c\right]-\mathbb{E}\left[\sum_{0\leq t\leq T_n}e^{-rt}c\Delta I_t\right]. \tag{A. 3}$$

It follows from $I_t^c=I_t-\Sigma_{0\leq s\leq t}\Delta I_s$ that :

$$\mathbb{E}[e^{-rT_n}\phi(K_{T_n})]\geq\phi(k)-\mathbb{E}\left[\int_0^{T_n}e^{-rt}\pi(K_t)dt+\int_0^{T_n}e^{-rt}c\,dI_t\right]. \tag{A. 4}$$

Taking $\lim_{n\to\infty}$ and using (3. 14) and the dominated convergence theorem yields :

$$\phi(k)\leq\mathbb{E}\left[\int_0^{\infty}e^{-rt}\pi(K_t)dt+\int_0^{\infty}e^{-rt}c\,dI_t\right]=J(k;I). \tag{A. 5}$$

From the arbitrariness of I, we have :

$$\phi(k)\leq\inf_{I\in\mathcal{A}}J(k;I)=V(k), \tag{A. 6}$$

which completes the proof of (I).

(II) For $k>\underline{k}$, from Lemma 3. 1, I^* is continuous for all $k>\underline{k}$ and increases only when $K^*=\underline{k}$. Then, for $I=I^*$ (A. 5) becomes the equality :

$$\phi(k)=\mathbb{E}\left[\int_0^{\infty}e^{-rt}\pi(K_t)dt+\int_0^{\infty}e^{-rt}c\,dI_t\right]=J(k;I^*)=V(k). \tag{A. 7}$$

For $k \leq \underline{k}$, it follows from Lemma 3. 1 that :

$$V(k) = c(\underline{k} - k) + V(\underline{k}). \tag{A. 8}$$

From (A. 7), we have $\phi(\underline{k}) = V(\underline{k})$. From the continuous property of $\phi(k)$, it follows that $\phi(\underline{k}) = d$. Thus, for all $k \leq \underline{k}$ we have :

$$V(k) = c(\underline{k} - k) + \phi(\underline{k}) = \phi(k). \tag{A. 9}$$

This completes the proof of (II).

References

Abel, A. B. and J. C. Eberly (1996), Optimal Investment with Costly Reversibility, *Review of Economic Studies,* 63, 581–593.

Dumas, B. (1991), Super Contact and Related Optimality Conditions, *Journal of Economic Dynamics and Control,* 15, 675–685.

Farzin, Y. H. and P. M. Kort (2000), Pollution Abatement Investment When Environmental Regulation is Uncertain, *Journal of Public Economic Theory,* 2, 183–212.

Guo, X. and H. Pham (2005), Optimal Partially Reversible Investment with Entry Decision and General Production Function, *Stochastic Processes and their Applications,* 115, 705–736.

Harrison, J. M. and M. I. Taksar (1983), Instantaneous Control of Brownian Motion, *Mathematics of Operations Research,* 8, 439–453.

Merhi, A. and M. Zervos (2007), A Model for Reversible Investment Capacity Expansion, *SIAM Journal of Control and Optimization,* 46, 839–876.

Lin, T. T., C.-C. Ko and H.-N. Yeh (2007), Applying Real Options in Investment Decisions Relating to Environmental Pollution, *Energy Policy,* 35, 2426–2432.

Perman, R., Y. Ma, J. McGilvray and M. Common (2003), *Natural Resource and Environmental Economics* 3rd ed., Person Education, Harlow.

Pham, H. (2006), Explicit Solution to an Irreversible Investment Model with a Stochastic Production Capacity, in : Kabanov, Y, R. Lipster and J. Stoyanov (eds.), *From Stochastic Calculus to Mathematical Finance : The Shiryaev Festschrift,* 547–566, Springer-Verlag, Berlin.

Pindyck, R. S. (2000), Irreversibilities and the Timing of Environmental Policy, *Resource and Energy Economics,* 22, 233–259.

Rogers, L. C. G. and D. Williams (2000), *Diffusions, Markov Processes and Martingales : Volume 2 : Ito Calculus,* 2nd ed., Cambridge University Press, Cambridge, U.K.

Saltari, E. and G. Travaglini (2011), The Effects of Environmental Policies on the Abatement Investment Decisions of a Green Firm, *Resource and Energy Economics,* 33, 666–685.

Yang, R-C. and K-H. Liu (2004), Optimal Singular Stochastic Problem on Harvesting System, *Applied Mathematics E-Notes,* 4, 133–141.

Zhao, J. (2003), Irreversible Abatement Investment under Cost Uncertainties : Tradable Emission Permits and Emission Charges, *Journal of Public Economics,* 87, 2765–2789.