Energy Detection using Pseudo BER based SNR Estimation Scheme in Cognitive Radio

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In this paper, a modified energy detection scheme using the pseudo bit error rate (BER) based signal-to-noise ratio (SNR) estimation is investigated. In the proposed scheme, the pseudo BER is used to estimate the SNR of the primary signal for the modified energy detection scheme at the secondary users. We have shown that with the assistance of the pseudo BER based SNR estimation, this modified energy detector outperforms traditional energy detector under the noise uncertainty in the run-time. In addition, the simulation results show that the proposed scheme can achieve the required detection performance and reliably alleviate the SNR wall effect as well.

Key words : Cognitive radio, Energy detection, Pseudo BER, SNR estimation

1. Introduction

Due to the increasing demands of the wireless devices and services, the traditional static spectrum allocation strategies are suffering inefficiencies in recent years. As the result, the Federal Communications Commission (FCC) sets about opening the TV bands and developing new policies intended for the unlicensed wireless devices to opportunistically access the vacant frequency bands [1]. Cognitive radio (CR), which has been introduced in [2], has emerged as a potential technology for those unlicensed wireless devices to reform the existing spectrum utilization allocation in an opportunistic manner. Accordingly, the CR devices are enabled to dynamically access the unused spectral holes whereas they are required to immediately vacate the occupied spectrum bands once the primary users (PU) transmission is detected, which demands the continuous spectrum sensing of the utilization status at the CR devices [3]. Intuitively, the spectrum sensing is definitely an essential and critical functionality that enables the CR devices to reutilize the idle spectrum bands without causing harmful interference to the incumbent wireless services.

As both the effectively sensing of the spectral holes and reliably detecting of the weak primary signals of possibly different types are important for the spectrum sensing in the CR, numerous detection methods have been researched, which can be classified into three broad categories: the energy detection [4], the matched filtering detection [5] and the cyclostationary detection [6]. Energy detector is recognized as the optimum detector if the prior knowledge of the primary signals is unknown at the CR receiver. On the other hand, for the primary signals of the known deterministic pattern (e.g., pilot, preamble, or training sequence), the optimal detector should be the matched filtering detection. However, perfect knowledge of the primary signals is required by the matched filtering, which would be impractical with the realistic limited cooperation between the PU and the CR users. The cyclostationary detection which exploits the built-in periodicity of the primary signals for more accurate detection, is mathematically intractable and requires the Monte-Carlo method to identify the optimal thresholds.

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A more detailed survey of the spectrum sensing techniques for the CR users can be found in [5] and the references therein.

In virtue of the hardware simplicities, the energy detection is considered not only the most common way but also the generic spectrum sensing methods as no prior knowledge is required at the CR detector. Previous studies on the energy-based detection have primarily focus on the detection performance in various static scenarios where the background noise is assumed to remain the same. However, the noise level varies with the time due to the temperature variations, the ambient interference, etc., which yields the noise uncertainty [7]. Thus, the actual performance of the energy detection would deviate significantly from the theoretic analysis because of its sensitiveness to the noise uncertainty. This limitation of the sensitiveness to the noise uncertainty is a so-called *signal-to-noise ratio* (SNR) *wall* effect which is defined [8] as the minimum SNR value below which the desired performance could not be attained even with the arbitrarily long sensing period. The cross-correlation based energy detection (ED) [9] has been suggested to alleviate the SNR wall effect. Furthermore, Mariani [10] has even revealed that the intrinsic cause of the SNR wall is due to the insufficient estimation of the noise power and the maximum likelihood (ML) estimation is applied for the improvements. However, those previous studies mainly concentrated on the performance enhancements while the system complexities and the detection durations were traded for that. Since the idea shows that the noise power is changing all the time and the SNR wall is inevitable, it motivates our work in this paper. We proposed an SNR estimation method based on a pseudo bit error rate (BER) for the modified ED scheme and then studied the detection performance in a dynamic manner while the system simplicities and the sensing durations are both considered.

The paper is organized as follows. In section 2, we describe the traditional energy detection for the CR devices and introduce the SNR wall problem. In section 3, we develop the pseudo BER based SNR estimation for the modified ED scheme, which seeks to minimize the average detection error rate (DER) during the whole time. Simulation results and discussion are provided in section 4. Finally, the paper is concluded in section 5.

2. Problem Formulation

2.1 *Signal model*

Consider the detection of the primary signals with a zero-mean additive white Gaussian noise (AWGN) at the CR users. A binary hypothesis testing is performed at the n-th time instant to identify the presence or the absence of the active PUs as

$$\mathcal{H}_0 : y(n) = w(n) \tag{1}$$

$$\mathcal{H}_1 : y(n) = w(n) + s(n) \tag{2}$$

where $\mathcal{H}_0$ stands for the absence of the primary signal $s(n)$ at the given time instant n, i.e. the received signal $y(n)$ contains only the noise $w(n) \sim \mathcal{CN}(0, \sigma^2)$, and $\mathcal{H}_1$ represents the presence of the primary signal $s(n)$ coexisting with the noise $w(n)$ at the given time instant n. Here, σ^2 is the noise power. Here, we define the average SNR as

$$\gamma = \frac{P}{\sigma^2}, \text{ where } P = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} |s(n)|^2 \tag{3}$$

The detection performance is always evaluated in terms of two probabilities: the probability of the detection P_d and the probability of the false alarm P_f

$$P_\mathrm{d} = \Pr(Y > \lambda \mid \mathcal{H}_1) \tag{4}$$

$$P_\mathrm{f} = \Pr(Y > \lambda \mid \mathcal{H}_0) \tag{5}$$

Here, the P_d is the probability while detecting a primary signal under the hypothesis $\mathcal{H}_1$ and the P_f is the probability of the false alarm under the hypothesis $\mathcal{H}_0$. Y is a decision statistic and λ is the corresponding test threshold for the each decision. For every spectrum sensing at the CR devices, the P_f should be kept as small as possible in order to prevent underutilization of the transmission opportunities while a large P_d is necessary and implies a higher chance that the CR devices detect the presence of primary signals.

The decision statistic for the traditional energy detector can be written as:

$$Y = \sum_{n=1}^{N} |y(n)|^2 \quad (6)$$

where N is the sensing length corresponds to the number of the observations. From the assumption above, the decision statistic Y follows the chi-square distribution with a degrees of freedom N in the case of $\mathcal{H}_0$ and the non-centralized chi-square distribution with a degrees of freedom N and a non centralized parameter γ in the case of $\mathcal{H}_1$:

$$Y \sim \begin{cases} \chi_N^{\ 2}, & \mathcal{H}_0 \\ \chi_N^{\ 2}(\gamma), & \mathcal{H}_1 \end{cases} \quad (7)$$

Observe from the Eqs. (4), (5) and (7), the decision threshold λ and the sensing length N can be chosen by finding an optimal balance between the desired P_d and P_f. For simplicity, the inverse function of the P_d can be approximated to be the inverse Q-function if N is sufficiently large due to the central limit theorem (CLT). And the lower bound of the sensing length can be obtained to be $N_{min}=2[Q^{-1}(P_f)-Q^{-1}(P_d)]^2\gamma^{-2}$ by eliminating the optimal threshold [8)]. $Q^{-1}(\cdot)$ is the inverse Q-function. We note that if the knowledge of the noise is known in advance, despite the arbitrarily low SNR γ, the desired performance can be obtained by appropriately choosing the optimal sensing length and the threshold.

2.2 *Problem Formulation*

However, it is known to all that the power of the background noise is non fixed value but an uncertain aggregation of various sources like the thermal noise, the leakage of signals from the other bands due to the receiver nonlinearity. Owing to that, the selection of the threshold, which would lead to a trade-off between the probabilities pair of P_d and P_f, could not be absolutely optimal.

Let consider the noise power σ^2 distributed in an noise uncertainty interval $[\sigma_n^2/\rho, \rho\sigma_n^2]$, where the σ_n^2 is the nominal noise power and the $\rho > 1$ is the parameter indicating the uncertainty degree. Accordingly, the desired detection performance, expressed as the probabilities pair of (P_d^*, P_f^*), should be satisfied with all the σ^2 in the given uncertainty interval that

$$P_d^{\ *} \le \min_{\sigma^2 \in [\sigma_n^{\ 2}/\rho, \rho\sigma_n^{\ 2}]} P_d \quad (8)$$

$$P_f^{\ *} \ge \max_{\sigma^2 \in [\sigma_n^{\ 2}/\rho, \rho\sigma_n^{\ 2}]} P_f \quad (9)$$

The probabilities pair (P_d^*, P_f^*) relates to the desired detection performance intuitively.

Accordingly, the uncertainty interval for the SNR γ can be expressed as $[\gamma_n/\rho, \rho\gamma_n]$ derived from the noise uncertainty interval and the $\gamma_n = P/\sigma_n^2$ is the nominal SNR value corresponding to the nominal noise power σ_n^2.

By the approximation and the simplification, the lower bound of the sensing length is given as [8)]

$$N_{\min} = 2[Q^{-1}(P_f^{\ *}) - Q^{-1}(P_d^{\ *})]^2[\gamma_n - (\rho - 1/\rho)]^{-2} \quad (10)$$

Clearly, as the $\gamma_n \rightarrow (\rho\text{-}1/\rho)$, the $N_{min} \rightarrow \infty$ and $\gamma_{wall} = \rho\text{-}1/\rho$ has been called the "SNR Wall" in [8]. To be exact, the required performance could not be achieved under certain noise uncertainty even with the arbitrarily long sensing length under. In other words, the detection performance of the traditional ED is sensitive to the noise uncertainty, which makes it less robust, especially at low SNR regime.

In order to alleviate the SNR wall effect, the noise uncertainty should be mitigated; alternatively, the SNR uncertainty should be mitigated. Our objective in this paper is to propose an SNR estimation method at a run-time based on the pseudo BER for a modified ED scheme, and investigate the adaption of the thresholds and sensing lengths dynamically, which could enhance the detection performance with certain constraints on the probability pair (P_d^*, P_f^*).

3. Pseudo BER based SNR Estimation for Energy Detection

In the previous section, it has been established that the detection probability is the function of the SNR level, the sensing length N and the threshold λ. Inspired by the exclusive relationship between the P_d and the γ, in this section, we apply a pseudo BER based SNR estimation for the modified ED scheme. Within the scheme, we can estimate the SNR γ frame by frame and then adapt the threshold λ according to the estimated SNR, which might diminish the SNR uncertainty, alleviate the SNR wall effect and improve the sensing reliability as well.

3.1 *Frame structure*

Figure 1 shows the frame structure designed for the modified ED scheme using the pseudo BER based SNR estimation. First of all, each frame consists of one sensing slot and one decision slot, the same with the traditional frame structure. Follow the sensing slot at the start of each frame; the decision is made for the rest of the frame according the sensing slot. Differences from the existing frame structure are:

- Under the n-th decision $\widetilde{\mathcal{H}}_0^n$: CR user starts the transmission during the n-th decision slot.
- Under the n-th decision $\widetilde{\mathcal{H}}_1^n$: CR user divides the n-th decision slot into K sub-slots and repeats the sub-sensing within each sub-slot.
- The i-th sub-sensing in the n-th decision slot: The output sequence at the i-th sub-slot is 1 for the sub-decision $\widetilde{\mathcal{H}}_1^n(i)$ and 0 for the sub-decision $\widetilde{\mathcal{H}}_0^n(i)$ with each modified thresholds.

where $\widetilde{\mathcal{H}}_0$ and $\widetilde{\mathcal{H}}_1$ denotes the decision assuming the absence and the presence of the active PU, respectively.

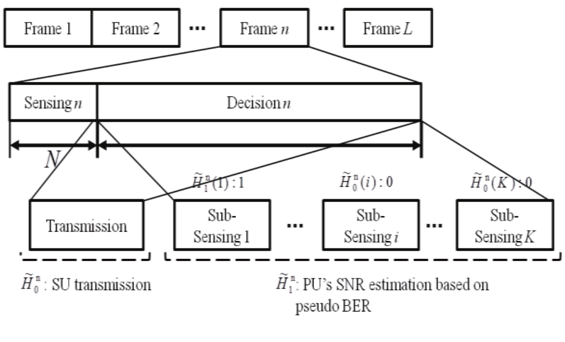

Fig. 1 Frame structure of pseudo BER based SNR estimation for energy detection.

3.2 *Pseudo BER based SNR estimation*

As has been argued, the exact SNR value should be applied to compute the optimal threshold and sensing length for the desired detection performance. However, the SNR value varies and the detection performance is unlikely satisfying with the fixed threshold and sensing length. We consider an SNR estimation method based on the pseudo BER and expect to update the threshold and the sensing length frame by frame with the run-time estimation.

The idea is derived from the exclusive relationship between the SNR and the error rate. It is intuitive to infer the instantaneous SNR from the error rate and eventually ease the SNR uncertainty. However, as a matter of fact, the true copy of the signal is unavailable at the receiver under the operating condition, not to mention the accurate error rate. The pseudo error rates [11] were firstly obtained with the modified thresholds and then extrapolated to determine the actual error rate. Further, the performance of the on-line pseudo error monitoring is detailed in [12] which shows that the intentionally degraded error rates overcome the short- comings of the long evaluation time and the interruption to the data traffic while monitoring the actual error rate. In this paper, we propose the SNR estimation method based on the pseudo BER by exploiting the error monitoring method as described in [13].

In accordance with the frame structure in the Fig.1, the proposed SNR estimation method based on the pseudo BER is illustrated in Fig. 2. The sub-sensing is performed if and only if the primary signal has been detected at the beginning sensing slot of the current

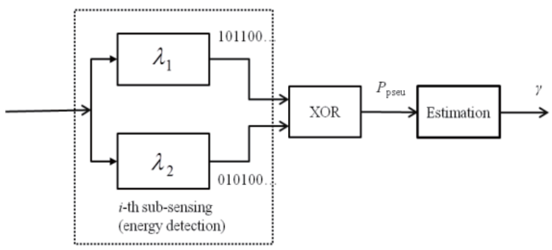

Fig. 2 Pseudo BER based SNR estimation scheme.

frame. In the Fig.2, each sub-slot includes two sub-sensing branches with two modified thresholds respectively, λ_1= (1-α) λ and λ_2= (1+α) λ. Here, the λ represents the normal threshold for the previous sensing slot and the α is the parameter for some offset on the normal threshold. Correspondingly, each branch outputs a sequence in which a 1 is for the sub-decision $\widetilde{\mathcal{H}}_1^n(i)$ or a 0 for the sub-decision $\widetilde{\mathcal{H}}_0^n(i)$ at the *i*-th sub-slot. After that, the exclusive OR (XOR) operations are executed on the decision sequences to count the disagreements for all the *K* sub-slots. Hence, the pseudo BER P_{pseu} is obtained. It can easily be shown that the P_{pseu} can be expressed mathematically by the probabilities of detection with different offset thresholds as

$$\begin{aligned} P_{\mathrm{pseu}} &\le \Pr(Y > \lambda_1 | \widetilde{\mathcal{H}}_1^n) - \Pr(Y > \lambda_2 | \widetilde{\mathcal{H}}_1^n) \\ &= \Pr((1-\alpha)\lambda < Y < (1+\alpha)\lambda | \widetilde{\mathcal{H}}_1^n) \end{aligned} \tag{11}$$

It has been demonstrated in the references that the pseudo BER plotted against SNR yields a curve similar in shape to a BER vs. SNR curve but with larger value. The desired relationship between the P_{pseu} and the P_{d} can thus be defined as

$$\log P_{\mathrm{pseu}}(\gamma) = M + [1 - \log \hat{P}_{\mathrm{d}}(\gamma)] \tag{12}$$

where the parameter $M > 0$ is the "gain" of the pseudo BER over the actual BER relate to the SNR value. The caret over the quantity indicates an estimation of the quantity.

The general steps carried out for the pseudo BER based SNR estimation can be generalized as follows: while the decision $\widetilde{\mathcal{H}}_1^n$ is assumed at the sensing slot, start the sub-sensing at the decision slot; then take the XOR operations on the decision sequences and the pseudo BER P_{pseu} is thus obtained; estimate the probability of detection $\hat{P}_{\mathrm{d}}$ from Eq. (12); find out the estimated $\hat{\gamma}$ eventually from the inverse function of the P_{d} in virtue of the statistic characteristics or just simply search within a lookup table.

3.3 *Energy detection using SNR estimation*

With the proposed scheme, the sub-sensing is carried out after the assertion of the active PU within the current frame and the pseudo BER based SNR estimation follows to update the estimated SNR for the subsequent detection frame by frame. Consider the actual dynamic scenarios with the noise uncertainty, in spite of the fluctuations, the SNR level would maintain during several frames.

As mentioned before, both the probabilities P_{d} and P_{f} are essential to the spectrum reutilization of the CR. In order to evaluate the detection performance for the CR, we firstly define the detection error rate (DER) as

$$P_{\mathrm{e}}(\gamma) = [1 - P(\mathcal{H}_1)] * P_{\mathrm{f}} + P(\mathcal{H}_1) * (1 - P_{\mathrm{d}}) \tag{13}$$

where $P(\mathcal{H}_1)$ is the prior probability of an active PU relative to the spectrum utilization status practically. The optimization criterion for the detection performance can be expressed as

$$\begin{aligned} &\min_{N,\lambda} P_{\mathrm{e}}(\hat{\gamma}) \\ &s.t. P_{\mathrm{f}} \le P_{\mathrm{f}}^* \text{and} \quad P_{\mathrm{d}} \ge P_{\mathrm{d}}^* \end{aligned} \tag{14}$$

where the P_{d}^* and the P_{f}^* are the desired probabilities. Based on the estimation $\hat{\gamma}$, given the desired pair of probabilities (P_{d}^*, P_{f}^*), we could always find the optimal sensing length *N* and the optimal threshold λ for the subsequent frame according to Eq. (14), which would undoubtedly minimize the DER. Thus, the DER might be minimized with the approximated SNR level based on the proposed scheme and meanwhile the SNR wall effect could be alleviated for the reduction of the noise uncertainty.

4. Simulation and Results

In this section, we evaluate the proposed pseudo BER based SNR estimation scheme for the modified ED scheme. Given the target probabilities P_{d}^* =0.9 and P_{f}^* =0.1, the initial sensing length and the threshold are assumed to be the optimal selection for the nominal γ_{n} at the very beginning.

4.1 *SNR wall*

We firstly confirm the SNR wall effect owing to the SNR uncertainty, viz. the noise uncertainty on the traditional ED scheme. Figure 3 indicates the SNR wall effect with three uncertainty degrees. Firstly, we note that, to achieve the required performance, the traditional ED should increase the sensing length as the nominal SNR level decreases. Secondly, given the uncertainty degree X=10log$_{10}\rho$ equals to 1 dB, 0.1 dB and 0.00 1 dB, the SNR walls locate around -3.3 dB, -13.4 dB and -33.4 dB, respectively. Especially with the uncertainty degree 1 dB, the necessary sensing length should approach to infinity as the nominal SNR is about -3.3dB, which is called as the SNR wall. We can also find that, with the less noise uncertainty, the SNR wall would be smaller and the traditional ED can be satisfying even within the lower SNR regime. On the contrary, the larger noise uncertainty is harmful for the traditional ED and alleviating the uncertainty should be necessary and helpful for the traditional ED.

Without loss of generality, in the following simulations, the noise uncertainty degree we choose to be X=1dB and the SNR wall should be around -3.3 dB accordingly.

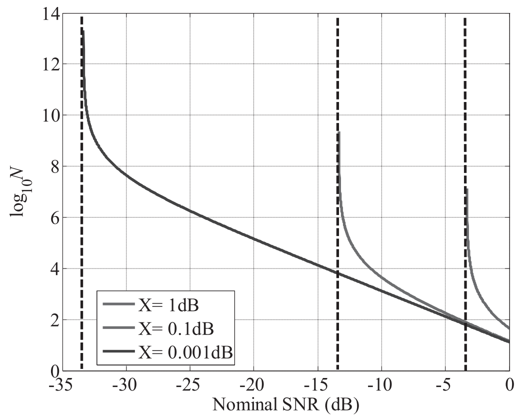

Fig. 3 SNR wall with different uncertainty degree *X*.

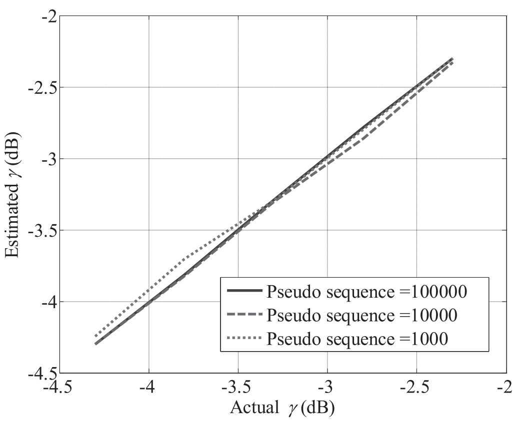

Fig. 4 SNR estimation performance.

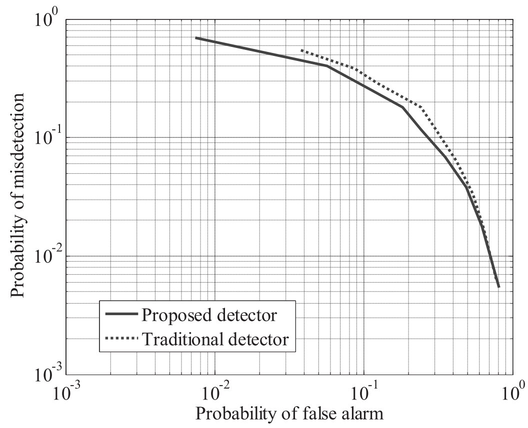

Fig. 5 ROC of the detection performance.

4.2 *Pseudo BER based SNR estimation*

Figure 4 shows the performance of the pseudo BER based SNR estimation. For the given offset parameter α =0.2, the detection probability $\hat{P}_{\mathrm{d}}$ could be well determined by the P_{pseu} with the Eqs. (12). The estimated $\hat{\gamma}$ can thus be obtained by calculating the inverse function of the P_{d}. Figure 4 shows the relationship between the estimated $\hat{\gamma}$ and the actual γ with the lengths of pseudo sequence increasing from 1000 to 100000. It is clear that the longer sequence realizes the better estimation performance. However, the pseudo sequence could not be very long as the estimation is executed during one frame and certainly time limited. We employ here the sequence length to be 1000, which is both applicable and accurate enough.

4.3 *Detection error rate optimization*

Eventually, we will give some simulation results for the scheme evaluation. The simulation parameters are: the prior probability of the active PU P($\mathcal{H}_1$)=0.7, and the offset parameter α=0.2. Figure 5 shows the comparison of the proposed scheme and the traditional scheme in term of the receiver operating characteristic (ROC) curve. The detection performance of the modified ED has been intuitively improved with the proposed scheme over the traditional ED.

Moreover, Fig. 6 compares the DERs between the proposed scheme and the traditional ED scheme. With the uncertainty degree X=1 dB, the sensing length N is adaptive for the proposed scheme ranging from 60 to 140 whereas 500 for the traditional ED. Most of the DERs for the proposed scheme have been found below 0.1, which is the preset target DER that can be calculated in advance. However, the traditional detector acts much worse with the same noise uncertainty as most of the DERs far exceed the target DER within the SNR uncertainty interval [-4.3 dB, -2.3 dB]. The comparison of the empirical CDFs is given in Fig.7. It shows that more than 80% simulation results of the proposed scheme are satisfied to be lower than target DER = 0.1 under the given noise uncertainty. Whereas almost half the simulation results of the traditional detector suffers from the poor performance (The DER is large than 0.1 because of the noise uncertainty). This means that the pseudo BER based SNR estimation has alleviated the SNR wall effect on the ED greatly and desirable performance can be achieved.

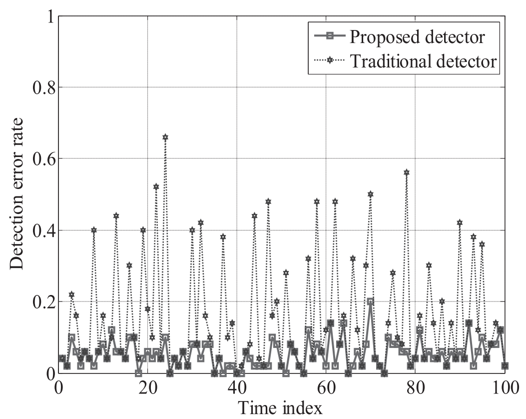

Fig. 6 Comparison of the detection performance.

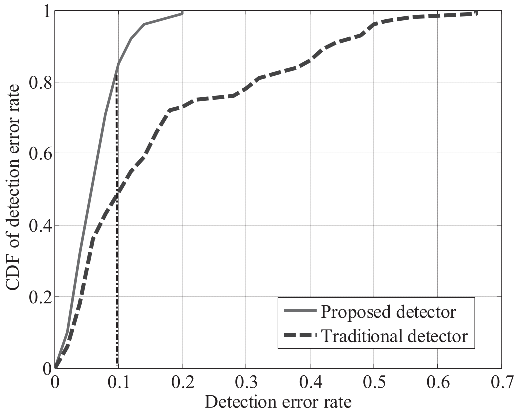

Fig. 7 Comparison of the empirical CDFs for detection error rates.

5. Conclusions

In this paper we proposed a modified energy detection scheme using the pseudo BER based SNR estimation for the CR users with certain constraints on the probability of detection and the probability of false alarm. An optimization method for the threshold adaptation has been applied to combat with the SNR wall effect. The proposed scheme is evaluated and its effectiveness has been confirmed within the given noise uncertainty interval. In particular, as a straightforward transformation of the traditional scheme, the proposed scheme manages to achieve the required performance even within short sensing duration.

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