

《研究ノート》

A Note on a Real Option and Game-theoretic Approach toward a Valuation of GHG Emission Rights in Climate Change

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1. Introduction

This paper focuses on a valuation of GHG (Greenhouse Gas) emission rights with a real option and game-theoretic approach.¹

Environmental destruction of the earth, such as global warming, is our important problem (Gore 2006). Toward this problem, it is said that GHG emission rights trading would be one of the most useful methods that might solve it, because emission rights trading would strike a balance between economical activity and ecology movement.²

Indeed emission rights trading have much possibility to solve the problem, and there are some arguments how we apply it relevantly in reality and how we think of the nature of emission rights.³ But these arguments seem to be not matured ; in particular, we have no consensus on a price theory of emission rights created in negotiated transactions such as CDM (Clean Development Mechanism) projects. This is the reason why we think of a price theory of emission rights.

We think of this problem with a real option and game-theoretic approach and we focus on CERs in negotiated transactions. This paper has two unique points. First, this paper has considered interactions of participants in emission rights trading and situations in negotiated transactions with a game-theoretic approach. Conrad (1997), Lambie (2009) and Pindyck (2000, 2002) have arguments of emission rights with real option approach, but they have not considered interactions of participants in emission rights trading and situations in negotiated transactions. On contrast, this paper has considered them.

Second, this paper focuses on CERs and processes in which CERs are created. CERs are

1 There are three types of GHG emission rights ; AAU (Assigned Amount Unit), ERU (Emission Reduction Unit) and CER (Certified Emission Reduction). This paper especially focuses on CER.

2 See Gibbons (1992) for game theory, Dixit and Pindyck (1994) for real option approach, and Dixit and Pindyck (1994, chapter. 9) for real option and game-theoretic approach.

3 For example, you can see this type of research in environmental economics and experimental economics.

created by CDM projects, and what important point in CDM projects is that they are made by negotiated transactions between a firm in a developed country and an underdeveloped country. Sakagami (2007) have arguments of emission rights with a real option and game-theoretic approach, but it has not considered this point. On Contrast, this paper focuses on this point.

In section 2, a two-players-model is presented. In section 3, we consider the equilibria in this model. In section 4, we sum up this model.

2. the model

2-1. Setup

This paper focuses on a valuation of emission rights ; in particular, CERs. We have considered interactions of participants in emission rights trading and situations in negotiated transactions with game-theoretic approach.

We assume two players in a CDM project ; a firm in a developed country (called ‘country A’ or ‘A’ in our model) and an underdeveloped country (called ‘country B’ or ‘B’ in our model). Country A is a Annex I in the Kyoto Protocol, and country B is a Annex II in the Kyoto Protocol. So country A has responsibility for the observance of the Kyoto Protocol. On the other hand, country B doesn’t have responsibility for the observance of the Kyoto Protocol.

Country A faces a decision-problem whether country A would participate in a CDM project with country B. On the other hand, country B also faces a decision-problem whether country B would participate in a CDM project with country A. Country A is the reader and country B is the follower in our model.

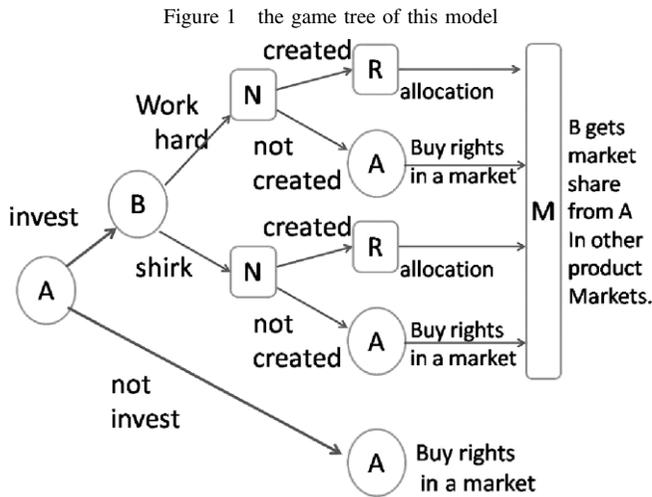
Timeline of this model is following table 1.

Table 1 Timeline

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|--|
| <ol style="list-style-type: none"> 1. Country A decides whether A would invest in a CDM project with B. To participate in the project, country A must pay I for it. I is defined as initial costs for the project. 2. When A invests, country B decides whether B would work hard or shirk in the CDM project. Country B must due the disutility of his work which is defined as ‘DU’. When B works hard, $DU=D$ (D is some fixed number), otherwise, $DU=0$.
When A doesn’t invest, A must buy emission rights in the markets in the 4 th period because of the limitation in the Kyoto Protocol. Unit price of emission rights in the 4 th period is P. 3. (When A invests.) ‘nature’ decides whether the CDM project with A and B would produce GHG emission rights. Probability that A and B get emission rights when B works hard is defined as $P(\text{CER}/\text{hard})$. Now we define $P(\text{CER}/\text{hard})$ as α. Probability that A and B wouldn’t get emission rights when B works hard is defined as $P(\text{not}/\text{hard})$, and we define $P(\text{not}/\text{hard})$ as $(1-\alpha)$. Probability that A and B would get emission rights when B shirks is $P(\text{CER}/\text{shirk})$. We define $P(\text{CER}/\text{shirk})$ as β. Probability that A and B wouldn’t get emission rights when B shirks is defined as $P(\text{not}/\text{shirk})$, and we define $P(\text{not}/\text{shirk})$ as $(1-\beta)$. We define $0 < \beta < 0.5 < \alpha < 1$. |
|--|

4. When the CDM project produces emission rights, A and B will share them with an allocation rule 'R'. R is defined as 'A : B = r : 1-r.' Now we define r as the ratio of this allocation, $0 \leq r \leq 1$. When the project doesn't produce them, A must buy emission rights in the market because of the limitation in the Kyoto Protocol. A price of emission rights in the market is P.
5. Country B gets some know-how of business from the project when A invests. This is because the know-how which A has in other business would be shared by A and B. In our model, the fact that B gets the know-how means that B would get market share in other products markets⁴ from A. This rate that B gets market share from A is 'M,' M is defined as 'A : B = 1-m : m.' Now we defined m as the ratio of the market share which B gets, $0 \leq m \leq 1$. Unit price of other product is defined as 1.

The game tree of this model is presented by figure 1.



2-2. Expected utilities

The expected utility of A is defined as EU_A . This consists of four factors ; allocation of CER, market share that B gets, initial investment, and P when A buy emission rights in a market.

$$EU_A = rCER - m - I - P \dots\dots\dots (1)$$

In reality, country A may compare the value of a CDM project (EU_A) with market prices that A can buy CER in a market before A makes a decision whether A invests in the first period. Now we define ρ as $1/(1 + \text{discounted rate})$. Now we defined that $\rho = 1$. For country A, the option value of postponement is just the value of the choice that A will buy emission rights in the market. Country A would compare a value of the project with the option value of postponement.

⁴ We assume that this market is monopoly for A.

【Observation 1】 : the condition that A participates in a CDM project

If $EU_A > P$, A would participate in a CDM project. If not, A would not participate.

Expected Utility of B is defined as EU_B . This consists of allocation of CER, market share that B gets, and DU.

$$EU_B = rCER + m - DU \quad \dots\dots\dots (2)$$

In reality, country B may compare the value when B works hard (this is defined as EU_B (*hard*)) with the value when B shirks (this is defined as EU_B (*shirk*)).

【Observation 2】 : the condition that B works hard in a CDM project

If $EU_B(\text{hard}) > EU_B(\text{shirk})$, B would work hard in a CDM project. If not, B would shirk in a project.

3. equilibria

3-1. three conditions

In this section, we consider three conditions to solve this problem backward.

First, we consider the condition of the maximization of EU_A when A invests in a CDM project. If we compare the EU_A when B works hard with the EU_A when B shirks, we will derive following condition.

【Corollary 1】 the condition of the maximization of EU_A when A invests in a project.

If $CER \geq \frac{m+I}{r}$, country A will maximize EU_A when B works hard. If $CER < \frac{m+I}{r}$, country A will maximize EU_A when B shirks.

【Proof】

See appendix 1.

Second, we consider the condition that B works hard or B shirks. From Observation 2, we will derive following corollary.

【Corollary 2】 the condition that B works hard or B shirks in a CDM project.

If $r \neq 1$ (that is, $0 \leq r < 1$), B would work hard in a CDM project. If $r = 1$, B would shirks in a project.

【Proof】

See appendix 2.

Corollary 2 shows that B shirks in a project when country A gets all emission rights.

Third, we consider the condition that A participates in a CDM project. From Observation 1, we will derive following corollary.

【Corollary 3】 the condition that A participates in a CDM project.

【Case 1】 When B works hard.
 If $CER \geq \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r}$, A would participate in a CDM project. If $CER < \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r}$, A would buy emission rights in a market.

【Case 2】 When B shirks.
 If, $CER \geq \frac{2-\beta}{\beta r} P + \frac{m+I}{\beta r}$ A would participate in a CDM project. If $CER < \frac{2-\beta}{\beta r} P + \frac{m+I}{\beta r}$, A would buy emission rights in a market.

【Proof】

See appendix 3.

3-2. equilibria

From three conditions, we derive following conditions (we write the decisions of country A and B as (A,B)).

【Lemma 1】 the condition of Nash equilibria in this model.

The condition that (A,B)=(invest in a CDM project, work hard) is Nash equilibria in this model is described as ;

$$CER \geq \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r}, \text{ and } r \neq 1.$$
 The condition that (A,B)=(invest in a CDM project,shirk) is Nash equilibria in this model is empty set.

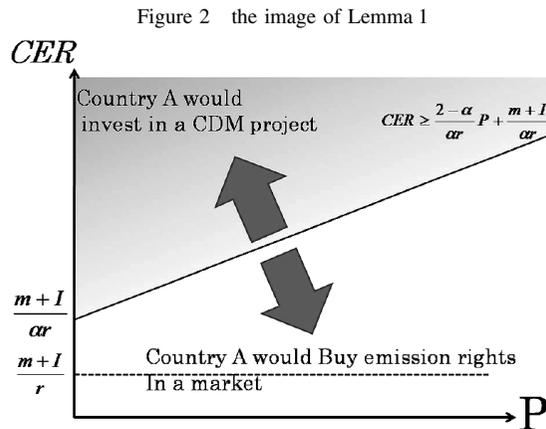
The condition that country A would buy emission rights is Nash equilibria in this model is described as ;

$$CER < \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r}$$

【proof】

See appendix 4.

We can describe Lemma 1 as following figure 2.



3-3. valuation

From Lemma 1, we can describe following theorem.

【Theorem】 the valuation of CERs from a CDM project.

If there is no arbitrage trade, the value of CERs from a CDM project would be described as following ;

$$CER = \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r} \quad (\text{s.t. } r \neq 1)$$

- ① If P increases, the value of CER will increase.
- ② If m increases, the value of CER will increase.
- ③ If r increases, the value of CER will decrease.
- ④ If α increases, the value of CER will decrease.

【proof】

See appendix 5.

③ and ④ are paradoxical conclusions.

4. Conclusion

The purpose of this paper is to consider a valuation of GHG emission rights, especially CERs from CDM projects with a real option and game-theoretic approach. There has been no study that tried to prove a valuation of them with a real option and game-theoretic approach.

We discovered Nash equilibria in a CDM project game, and we got a theorem of a valuation of CERs. The following results were obtained :

- (1) In a CDM project, an underdeveloped country shirks in a project when a firm in a developed country will get all emission rights in the contract ($r=1$).
- (2) a value of CERs from CDM projects is affected by following factors :
 - (a) a market price of emission rights (P),
 - (b) the ratio of allocation of CERs between an underdeveloped country and a firm in a developed country in CDM contracts (r),
 - (c) the ratio of the market share which an underdeveloped country gets from a firm in a developed country in other products markets because of know-how which an underdeveloped country gets in CDM projects (m),
 - (d) initial costs of CDM projects which a firm in a developed country must pay (I), and
 - (e) the probability that an underdeveloped country and a firm in a developed country get CERs when an underdeveloped country works hard in CDM projects (α).
- (3) the value of CERs from CDM projects will decrease when (b) or (e) increases.

※This work was supported by Grant-in-Aid for Young Scientists (B) of the Ministry of Education, Culture, Sports, Science and Technology.

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Appendix

Appendix 1 the proof of Corollary 1.

We think of EU_A if A invests.

【Case 1】 When B works hard.

$$EU_A = \alpha(rCER - m - I) + (1 - \alpha)(-m - I - P)$$

【Case 2】 When B shirks.

$$EU_A = \beta(rCER - m - I) + (1 - \beta)(-m - I - P)$$

When we compare both of them, we can get Corollary 1.

Appendix 2 the proof of Corollary 2.

We think of EU_B if A invests.

【Case 1】 When B works hard.

$$EU_B(\text{hard}) = \alpha((1 - r)CER + m - D) + (1 - \alpha)(m - D)$$

【Case 2】 When B shirks.

$$EU_B(\text{shirk}) = \beta((1 - r)CER + m - D) + (1 - \beta)(m - D)$$

From observation 2, the condition that B Works hard is $EU_B(\text{hard}) - EU_B(\text{shirk}) > 0$. We solve this.

$$\begin{aligned} & \alpha((1 - r)CER + m - D) + (1 - \alpha)(m - D) - \beta((1 - r)CER + m - D) - (1 - \beta)(m - D) > 0 \\ & \Leftrightarrow (\alpha - \beta)(1 - r)CER > 0 \end{aligned}$$

Now $\alpha > \beta$ and $CER > 0$, so we need the condition '1 - r > 0' when B works hard.

Appendix 3 the proof of Corollary 3.

From Observation 1, we will derive $EU_A > P$. We solve this.

【Case 1】 When B works hard.

$$EU_A = \alpha(rCER - m - I) + (1 - \alpha)(-m - I - P) > P$$

$$\Leftrightarrow CER \geq \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r}$$

【Case 2】 When B shirks.

$$EU_A = \beta(rCER - m - I) + (1-\beta)(-m - I - P) > P$$

$$\Leftrightarrow CER \geq \frac{2-\beta}{\beta r} P + \frac{m+I}{\beta r}$$

Appendix 4 the proof of Lemma 1.

From Corollary 1-3, we will describe the condition of Nash equilibria in this model as following figure.

		B	
		Work hard	shirk
A	Invest in a CDM project	$CER \geq \frac{m+I}{r}$ $r \neq 1 (0 \leq r < 1)$ $CER \geq \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r}$	$CER < \frac{m+I}{r}$ $r = 1$ $CER \geq \frac{2-\beta}{\beta r} P + \frac{m+I}{\beta r}$
	Buy emission Rights in a market	$CER < \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r} \text{ or } CER < \frac{2-\beta}{\beta r} P + \frac{m+I}{\beta r}$	

First, we think of (A, B)=(invest, work hard). Because of $\alpha < 1$, we will get the condition ' $\frac{m+I}{r} < \frac{m+I}{\alpha r}$ '. So we can rewrite the condition that (A, B)=(invest, work hard) is Nash equilibria in this model as following.

$$CER \geq \frac{2-\alpha}{\alpha r} P + \frac{m+I}{\alpha r}, \text{ and } r \neq 1. \quad \dots\dots\dots (3)$$

Second, we think of (A, B)=(invest, shirk). From the condition of 'r=1', we can rewrite the condition as following.

$$CER < m+I, CER \geq \frac{2-\beta}{\beta} P + \frac{m+I}{\beta} \quad \dots\dots\dots (4)$$

Because of $\beta < 1$, we will get the condition ' $\frac{m+I}{r} < \frac{m+I}{\beta r}$ '. So the condition of (4) is empty set.

Appendix 5 the proof of Theorem.

If there is no arbitrage trade, we can define ' $EU_A = P$.' From Lemma 1 and this condition, we can get Theorem.